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Forensic Engineering Analysis of Dynamic Forces Created by Pedestrians Impacting Plate Glass at Different Speeds

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Abstract

This paper examines the effect of a pedestrian inadvertently impacting a glass panel adjacent to a glass exit door. The glass panel was full length and unmarked, violating the local building code and building commissioner directives. The defense argued that the old building code, to which the college building was constructed, was "grandfathered" and there was no violation. Initial witness and plaintiff statements indicated that the plaintiff was running when he struck the panel, which shattered and caused the plaintiff to incur injuries. The plaintiff testified later, in deposition, that he was walking when he struck the glass. The court needed to establish liability percentage, which is common in civil cases, and it was necessary to determine at what pedestrian impact speed the glass panel would break. The analysis used Timoshenko's theory of vibration plates, Roark's stress strain formulas, ergonomics, and human factors to estimate the impact load and stresses on the glass panel that caused failure.

Keywords

Glass, vibration, building code, directives, differential equation, ergonomics, human factors

Case Description

The plaintiff had received a message that a family member had just been taken to the hospital, and he was descending the stairs of a state-owned college building. Adjacent to the double exit doors were full-length unmarked glass panels that were in-line with the stair route. The plaintiff reached out, arm fully extended, and pushed on the unmarked glass side panel, thinking it was the exit door. The glass panel was not made of safety glass (annealed glass), and his arm pushed at approximately the center of the panel. It broke in shards, causing severe cuts along his wrist and forearm. EMS was called, and the plaintiff was taken to the hospital. He stated to EMS and building security guards that he was running down the stairs, while on his cell phone, and thought that the panel was the door (see **Figure 1**).

Figure 1
The photo shows the view of the stairs, landing, exit doors, and glass panel that was impacted by the plaintiff as he was descending the stairs. The trash containers were not in front of the glass panel at the time of incident. The photograph was taken at the time of the inspection; therefore, the panel had been repaired.

Plaintiff Argument

The plaintiff engaged an expert engineer who claimed that the unmarked glass panel violated certain sections of Industrial Board of Appeals Chapter 1 Part 47 and, in particular, Part 47.8. The plaintiff's Expert cited codes from the current 2008 laws — the year the incident occurred — and wrote:

> "…which stated that the fixed glass panel was clear and gave the optical illusion as if it did

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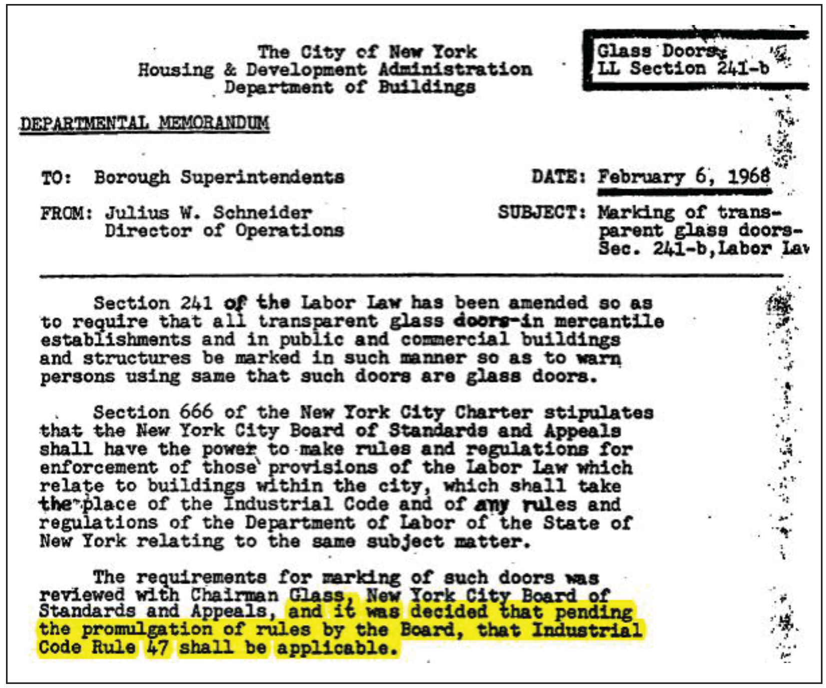
not exist, and gave the impression that there was a straight walking distance to the outer lobby. It should be noted that that transparent glass doors and fixed adjacent transparent glass sidelights shall be marked in two areas of the glass surface thereof. One such area shall be located at least 30 but not more than 36 inches and the other at least 60 but not more than 66 inches above the ground, or floor or equivalent surface below the door or sidelight."

"…that the New York State Labor Law Section 241-B requires that all transparent glass doors in mercantile establishments and in public and commercial buildings and structures shall be marked in such a manner as shall be calculated to warn persons using the same that such doors are glass doors."

After the plaintiff retained his lawyer, he cited in his deposition and all subsequent dialogs that he was "walking." Therefore, he retracted his earlier statement that he was running, and changed it to walking.

Compilation of Codes, Rules and Regulations of the State of New York, cited as 12 NYCRR47, Transparent Glass Doors in Mercantile Establishments and in Public and Commercial Buildings and Structures (see **Figure 2**).


The City of New York
Housing & Development Administration
Department of Buildings

Glass Doors
LL Section 241-b

DEPARTMENTAL MEMORANDUM

TO: Borough Superintendents

DATE: February 6, 1968

FROM: Julius W. Schneider
Director of Operations

SUBJECT: Marking of transparent glass doors-Sec. 241-b, Labor Law

Section 241 of the Labor Law has been amended so as to require that all transparent glass doors in mercantile establishments and in public and commercial buildings and structures be marked in such manner so as to warn persons using same that such doors are glass doors.

Section 666 of the New York City Charter stipulates that the New York City Board of Standards and Appeals shall have the power to make rules and regulations for enforcement of those provisions of the Labor Law which relate to buildings within the city, which shall take the place of the Industrial Code and of any rules and regulations of the Department of Labor of the State of New York relating to the same subject matter.

The requirements for marking of such doors was reviewed with Chairman Glass, New York City Board of Standards and Appeals, and it was decided that pending the promulgation of rules by the Board, that Industrial Code Rule 47 shall be applicable.

Figure 2
The marking of glass doors and side panels went into effect on Jan. 1, 1968.
The building commissioner's directive was made public on Feb. 6, 1968.

Defense Argument

The author researched the building department website and discovered that the building was constructed circa 1962 under the 1938 Building Code of the City of New York — where there was no requirement for glass markings on doors or side panels.

The defense attorney's theory of the case was that the unmarked glass side panel was "grandfathered" in the building code because the old building code was silent regarding glass panel markings. However, the building commissioner issued a directive in 1961, which became part of the labor law, requiring glass side panels adjacent to glass exit doors to be marked.

These directives were made public when issued and served as clarifications of the building code by the commissioner of buildings. The glass door and side panel markings directive went into effect on Jan. 1, 1968.

The Department of Labor industrial commissioner issued a directive as Part 47 of Title 12 of the Official

The directive did not support the defense attorney's theory because it required the building owner to mark the glass panels and had been in effect for at least 40 years.

The author agreed with the opposing expert that the building owners had not marked the panels, which was a violation. The author's initial suggestion was that the defense submit to the claim. However, on reading the case material, the author saw the change in oral evidence from "run" to "walk" and suggested to the defense (who did not realize the walk/run distinction) that an analysis be performed to compare the stresses on the glass between the impact of running and walking.

The author was permitted to perform an analysis regarding the stress on the glass panel for walking and running into the panel. This required engineering analysis to determine the magnitude of dynamic force necessary to break the glass panel and let the trier of fact apportion liability to the parties accordingly.

Case Material Reviewed

The author reviewed three photostats of photographs of the inner entranceway of the glass side panel where the event occurred, the verified bill of particulars with attachments, the opposing expert disclosure, and the report of the plaintiff's expert. That report included: photostats of photographs, original design engineering calculations for wind, and plans of the entranceway doors, mullions, connections, etc. Also reviewed were the fire department's EMS report, the safety department incident report, several depositions (including the plaintiff and building maintenance personnel), and references regarding human factors, ergonomics, and material properties of plate glass.

The author visited the location and took measurements/photographs. On the date of the inspection, the scene was not in the same condition represented in the photographs because the interior easterly glass panel had been repaired. Note that the glass panel was in direct line and direction of travel of the plaintiff descending the stairs (**Figure 1**).

Analysis Methods

The glass panel consisted of two ¼-inch-thick panels sandwiched together, as shown in **Figure 3**, which was adjacent to the exit doorway. The panel on the other exit doorway was the original glass, and the author was able to measure the thickness as well as the unsupported height and width of the panel. The glass panel appeared to be substantially fixed and supported around all edges of the panel, but the window frame could not be disassembled to ascertain the specific mounting and weather sealing method used. The analysis assumption was that the glass panel would behave as a membrane because the thickness relative to the shortest dimension was large — approximately 90 to 1, which is above the threshold of 80 to 1 for membrane analysis[1]. The analysis assumed that the membrane was flexible and infinitely thin, of uniform material and thickness, and that it would elastically stretch uniformly in all directions when deflected. Hence, if it failed, it would fail in tension[1,2]. The impact load was considered to be perpendicular to the plate surface. Testing of the pushing strength was estimated to be approximately 15% of the male's weight. Based on the plaintiff's weight of 180 pounds, that pushing force equals 27 pounds of force[3]. Another source of arm strength estimated 37 pounds of force[4]. Based on the average for the arm pushing force against the panel on sources, the author used 32 pounds of force

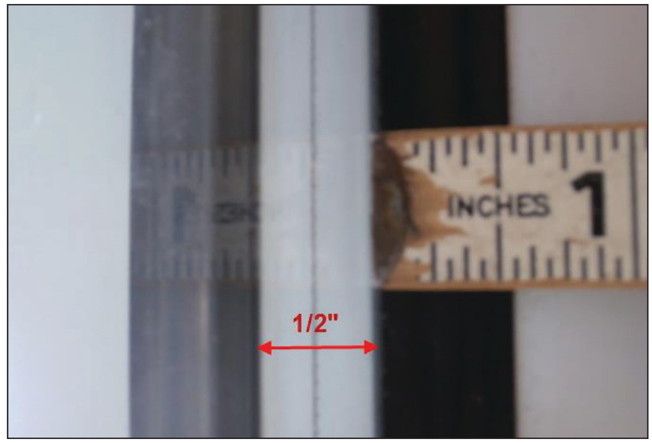


Figure 3
The photograph represents the measurement taken of the thickness of the sandwiched glass panels of ¼ inch each. The panel was the original installed glass panel similar to the one that the plaintiff impacted.

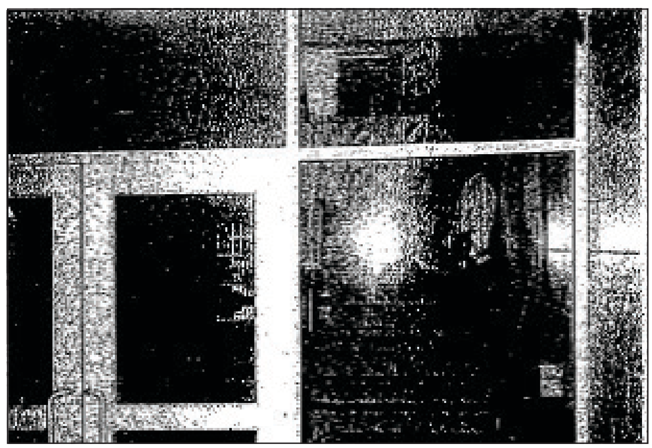


Figure 4
The poor photographic reproduction shows the glass panel as viewed from the outer lobby entrance. The shards of glass still remaining in the frame can be barely observed.

Figure 5
Poor photographic reproduction of the glass panel taken from the outer lobby to show the glass shards. The shards of glass remaining in the frame would indicate that the panel was substantially fixed within the frame.

for the load on the glass panel. See calculations in the **Appendix** for further analysis.

The author performed an analysis to determine the stress on the glass panel walking at 4 feet per second (fps), the design speed for walking per the Manual of Uniform Traffic Control Devices (MUTCD), the American Association of State Highway and Transportation Officials (AASHTO), and other studies that are referenced to determine the running speed of pedestrians descending stairs and then accelerating to a run or sprint. The average fast descending speed on stairs of pedestrians is approximately 2.3 fps. The average acceleration of a pedestrian reaching the ground floor landing while still in motion is approximately 0.2 g's — for a maximum of 1 second. This calculated to a pedestrian velocity, using the two velocity equations (one using a time and the other using distance). Using a time of 1 second, the velocity yields 8.7 fps; using a distance of 7 feet, the distance of the bottom step to the glass panel, yields 9.8 fps. The author averaged the two results and used 9.25 fps. This value of the plaintiff's speed was derived from various tests of pedestrians descending stairs and the acceleration of sprinters/runners from a walk to a run [5-12]. It should be noted that the speed of pedestrians is a function of slope of the stairs, dimension of the treads and risers, and the number of other pedestrians in the area. At the time of incident, there was a change in classes so the corridors and stairs were not empty, which may have affected the speed of the plaintiff.

As mentioned, the glass panel was treated as a thin plate or membrane. The dimensions of the glass plate were 81.75 inches in height, 45.5 inches in width, and two ¼-inch thicknesses. The thickness used for glass was calculated using the equivalent thickness of two ¼-inch panels into one equivalent thickness. The calculation was arrived at by equating the two ¼-inch panels using mass moment of inertia into the equivalent of one glass panel. The resulting equivalent thickness of a single glass panel was 0.50005 inches. Sliding between the two ¼-inch glass panels was not considered because the edges of both panels were substantially fixed and tightly sandwiched together in the frame, and the author was not able to compare this model to a sliding model. This assumption may be a source of error. See labeled mass moment of inertia calculations in the **Appendix**. Another potential source of error in "combining" these glass panels is that glass is a brittle material and will behave differently than a typical elastic-plastic deforming material. The glass panel was annealed glass, not safety glass, which would have broken in small pieces — not the shards that caused the serious cuts in the plaintiff's arm and wrist [13].

In order to determine the breaking stress on the glass panel, it was necessary to derive the dynamic impact force of the panel and apply the loads to a static model. Because the thickness of the glass panel was much smaller than the shortest dimension of the panel, the panel could be treated as a membrane. The force was dynamically applied. Using Timoshenko's "Vibration Problems in Engineering"[2], a 2^{nd} order differential equation was set up and solved using the initial conditions. The initial conditions were: (1) at time zero, the deflection of the panel was zero; and (2) at time zero, the change in velocity of the panel was due to the impact. Damping of the vibration was considered to be between 1% and 8% of the natural frequency. A damping vibration of 4% of the natural frequency was used based on the density and thickness of the glass panel [14].

The analysis of the model calculated the acceleration and force on the glass panel using an impact speed of 4 fps and 9.25 fps as the initial conditions. The force on the model glass panel was a pressure force at the mid-point of the panel derived from the estimated dynamically pressing of the hand on the panel. The estimated area of the hand applying the load was approximately 18 square inches. The acceleration was calculated, and the corresponding force was the equivalent static load applied to the glass panel. See graphs 1a, 1b, 2a, and 2b in the **Appendix**. Once the force was calculated, the load was placed in "Roark's Stress and Strain Formulas" for flat plates with fixed edges all around. Roark Formula criteria were that the plate was not stressed beyond its elastic limit[15]. Because glass is a brittle material, it would not be stressed beyond the elastic limit. As a comparison, the author compared the end conditions if the plate was simply supported and loaded identically. The method and calculations are attached in the **Appendix**.

Results

The outcome of the analysis for fixed edges resulted in a stress of approximately 2,565 psi for walking into the glass panel, which was less that the tensile failure stress of 5,000 psi for glass. The stress for running into the glass panel was approximately 5,932 psi, which exceeded the tensile failure stress of 5,000 psi for glass[16]. If the plate was simply supported on all edges, the results were 3,269 psi and 7,559 psi for walking and running, respectively. Under the same loading conditions, the simply supported edges would undergo higher stresses as compared to the fixed supported edges. One consideration that could not be accommodated was the potential effect on edge damping that could result from the use of rubber or other compliant weather sealing material between the windows and rigid frame. See the **Appendix** for the Roark calculation.

Conclusion

Within a reasonable degree of engineering certainty — and within the estimates and approximations stated — walking/running speeds greater than normal walking speeds would have broken the glass under the equivalent dynamic loading conditions. The fact that the plaintiff was running, as originally stated, resulted in an apportionment of liability by the trier of fact.

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Appendix

$Wt := 180 \cdot lb$ — Weight of male contacting glass panel.

$R_{arm} := 32\ lbf$ — Force the arm pushes on the glass panel per an average of various ergonomics estimates 3-6

Calculation of the running speed into the glass panel by the plaintiff using data from pedestrains studies of pedestrians descending stairs; and the acceleration of runners/sprinters off the start line adjusted for the non-sprinter/runner.

$v_{descend} := 2.3\ \frac{ft}{s}$ — Average stair descending pedestrian speed from reference Centre for Transport Studies, University College London,United Kingdom 7-14

$f_{acc} := 0.2$ — Estimated acceleration based on runners/sprinters off the start line as 1 g and adjusted for pedestrian already in motion and not in a competition.

$dist_{landing} := 7\ ft$ — Landing length from bottom stair to the glass panel.

$t_{acc} := 1\ s$ — Standard pedestrian acceleration time from start.

$v_{impact1} := v_{descend} + f_{acc} \cdot g \cdot t_{acc}$ — Impact speed using time of acceleration.

$$v_{impact1} = 8.73\ \frac{ft}{s}$$

$v_{impact2} := \sqrt{v_{descend}^{\ 2} + 2 \cdot f_{acc} \cdot g \cdot dist_{landing}}$ — Impact speed using the distance from the stair to the glass panel.

$$v_{impact2} = 9.77\ \frac{ft}{s}$$

$v_{impact} := \frac{v_{impact1} + v_{impact2}}{2}$ — Average velocity calculation.

$v_{impact} = 9.25\ \frac{ft}{s}$ — Use the average impact speed of the time and distance calculated.

Calculation of the equivalent thickness of a single glass panel from two 1/4" glass panels sandwiched and fixed all around the edges.

Using mass moment of inertia find the equivalent plate thickness of one plate in place of two 1/4" plates.

$t_1 := 0.25 \cdot in$ Plate thickness.

$L_w := 81.75 \cdot in$ Plate length (b).

$W_w := 45.5 \cdot in$ Plate width (a).

$$I_1 = 2 \cdot \left(\frac{1}{12} \cdot \left(\frac{\rho \cdot L_w \cdot W_w \cdot t_1}{g} \cdot W_w^{\,2} + \frac{\rho \cdot L_w \cdot W_w \cdot t_1}{g} \cdot \left(\frac{t_1}{2} \right)^2 \right) \right)$$

Moment of inertia for 2 - 1/4" plates moving the cg to the edge of the plate.

$$I_2 = \frac{1}{12} \cdot \frac{\rho \cdot L_w \cdot W_w \cdot t_2}{g} \cdot W_w^{\,2}$$

Moment of inertia of the equivalent plate.

$$\frac{1}{12} \cdot \frac{\rho \cdot L_w \cdot W_w \cdot t_2}{g} \cdot W_w^{\,2} = 2 \cdot \left(\frac{1}{12} \cdot \frac{\rho \cdot L_w \cdot W_w \cdot t_1}{g} \cdot W_w^{\,2} + \frac{\rho \cdot L_w \cdot W_w \cdot t_1}{g} \cdot \left(\frac{t_1}{2} \right)^2 \right)$$

$$\frac{1}{12} \cdot t_2 \cdot W_w^{\,2} = 2 \cdot \left(\frac{1}{12} \cdot t_1 \cdot W_w^{\,2} + t_1 \cdot \left(\frac{t_1}{2} \right)^2 \right)$$

Equating the moments of inertia.

$$t_2 = 2 \cdot \left(t_1 + \frac{12 \cdot t_1}{W_w^{\,2}} \cdot \left(\frac{t_1}{2} \right)^2 \right)$$

Equivalent thickness of plate.

$$t_2 := \left(t_1 + \frac{3 \cdot t_1^{\,3}}{W_w^{\,2}} \right) \cdot 2 \qquad t_2 = 0.50005\ in$$

This is the equivalent thickness of 2 - 1/4" plates sandwiched together and clamped (fixed) on all edges as one plate.

Orientation of plate glass where "b" in the long edge and "a" is the short edge.

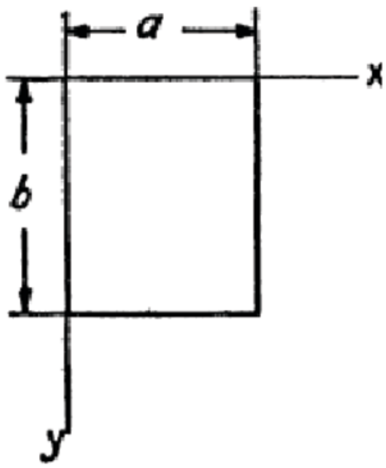


Figure 6.

Note: The properties of annealed glass were taken from various sources with an approximate average used in this analysis.

Specifications of glass panel.

$\rho := 2.2 \cdot \frac{gm}{cm^3}$	$\rho = 137.3\ \frac{lb}{ft^3}$	Density of glass.
$E := 10000000 \cdot \frac{lbf}{in^2}$	$E = 10000000\ psi$	Young's modulus for glass.
$L := 81.75 \cdot in$		Vertical dimension of glass panel.
$W := 45.5 \cdot in$		Horizontal dimension of glass panel.
$H := 0.50005 \cdot in$		Thickness of two quarter inch thickness of glass panel.
$W_g := L \cdot W \cdot H \cdot \rho$	$W_g = 147.8\ lb$	Weight of equivalent thickness glass panel.

Using momentum, calculate the velocity that the glass panel is subjected to as it sits edges fixed in the frame. Use 4 fps as estimated walking speed and 9.25 fps as running speed.

$W_b := R_{arm}$	$W_b = 32\ \mathit{lbf}$	Force of boy's arm striking glass panel.
$M_g := \frac{W_g}{g}$	$M_g = 4.6\ \frac{lb \cdot s^2}{ft}$	Mass of glass panel.
$M_b := \frac{Wt}{g}$	$M_b = 5.6\ \frac{lb \cdot s^2}{ft}$	Mass of boy striking glass panel.
$v_{b1} := 9.25 \cdot \frac{ft}{s}$	$v_{b2} := 4 \cdot \frac{ft}{s}$	Velocity boy strikes glass panel walking and running.
$v_{g1} := \frac{Wt}{(Wt + W_g)} \cdot v_{b1}$	$v_{g1} = 5.08\ \frac{ft}{s}$	Change in velocity of the glass running. From Linear Momentum.
$v_{g2} := \frac{Wt}{(Wt + W_g)} \cdot v_{b2}$	$v_{g2} = 2.2\ \frac{ft}{s}$	Change in velocity of the glass walking. From Linear Momentum.

From Linear Momentum:

$$m_b \cdot v_b + m_g \cdot v_b = m_b \cdot v_b' + m_g \cdot v_g'$$

$$m_b \cdot v_b = (m_b + m_g) \cdot v'$$

Since the initial velocity of the glass panel is zero; and both the boy and the glass panel achieve the same velocity at impact then the post velocities are the same.

$$v' = \frac{m_b}{(m_b + m_g)} \cdot v_b$$

v' is the velocity of the glass panel to be used as the initial condition at time zero.

Analyze the plate glass window using Timoshenko, "Vibration Problems in Engineering", 2nd Edition 5th printing, 1937, D. Van Nostrand Company, Section 70, Vibration of Plates, beginning on page 421. Figure 8.

$$\frac{\gamma \cdot h}{g} \cdot q''(t) + c \cdot q'(t) + \pi^4 \cdot D \cdot q(t) \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right) = 0$$

Differential equation for normal vibration from an applied force with damping term added to equation.

$$p = \pi^2 \cdot \sqrt{\frac{g \cdot D}{\gamma \cdot h} \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)}$$

Natural frequency of glass panel.

$$2 \cdot s = \frac{c \cdot g}{\gamma \cdot h}$$

Damping term which will be a function of the natural frequency term used further down in calculation.

$$q''(t) + 2 \cdot s \cdot q(t) + p^2 \cdot q(t) = 0$$

Equation in standard form.

$$q(t) = e^{r \cdot t} \qquad q'(t) = r \cdot e^{r \cdot t} \qquad q''(t) = r^2 \cdot e^{r \cdot t}$$

Trial solution.

$$r^2 + 2 \cdot r \cdot s + p^2 = 0$$

$$r_1 = -s + \sqrt{s^2 - p^2} \qquad r_2 = -s - \sqrt{s^2 - p^2}$$

$$q(t) = C_1 \cdot e^{\left(-s + \sqrt{s^2 - p^2}\right)} + C_2 \cdot e^{\left(-s - \sqrt{s^2 - p^2}\right)}$$

General solution to differential equation.

$$-{p_1}^2 = p^2 + s^2$$

$$q(t) = e^{-s \cdot t} \cdot \left(C_1 \cdot cos\left(-p_1 \cdot t\right) + C_2 \cdot sin\left(-p_1 \cdot t\right)\right)$$

$$t = 0 \qquad q(t) = 0 \qquad q'(t) = v$$

Initial conditions.

$$C_1 = 0$$

$$q'(t) = -e^{-s \cdot t} \cdot p_1 \cdot C_2 \cdot cos\left(p_1 \cdot t\right) - s \cdot e^{-s \cdot t}\ C_2 \cdot sin\left(p_1 \cdot t\right)$$

$C_2 = \frac{-v}{p_1}$

$q(t) = e^{-s \cdot t} \cdot \left(\frac{v}{\sqrt{p^2 - s^2}} \right) \cdot sin\left(\sqrt{p^2 - s^2} \cdot t\right)$ — Equation of vibration after solving differential equation.

$h := H$ — $h = 0.50005\ in$ — Thickness of two glass panels.

$v := 0.23$ — Poisson's ratio for glass.

$\gamma := 0.0806 \cdot \frac{lb}{in^3}$ — Glass panel density.

$D := \frac{E \cdot h^3}{12 \cdot (1 - v^2) \cdot g}$ — $D = 9168\ lb \cdot ft$ — Flexural rigidity of glass panel.

$a := W$ — $a = 45.5\ in$ — Short horizontal dimension of glass panel.

$b := L$ — $b = 81.75\ in$ — Long vertical dimension of glass panel.

$area_g := a \cdot b$ — $area_g = 26\ ft^2$ — Area of glass panel.

$m := 1$

$n := 1$

m & n are Integers for the lowest mode of vibration in Timoshenko equations. Vibration of panel (page 413)

$p := \pi^2 \cdot \sqrt{\frac{g \cdot D}{\gamma \cdot h}} \cdot \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$ — $p = 203\ \frac{1}{s}$ — Natural frequency of glass panel.

$T := \frac{2 \cdot \pi}{p}$ — $T = 0.031\ s$ — Period of glass panel.

$t := 0 \cdot ms, .1 \cdot ms .. \frac{T}{4}$ — Time of maximum vibration loading.

$\xi := 0.02$

Damping based on density of 1/2 inch glass as percentage of natural frequency.

$s := \xi \cdot p$ $\quad s = 4.054 \ \frac{1}{s}$

Damping term.

$$q_1(t) := e^{-s \cdot t} \cdot \left(\frac{v_{g1}}{\sqrt{p^2 - s^2}} \right) \cdot \sin\left(\sqrt{p^2 - s^2} \cdot t \right)$$

Exact solution of differential equation using initial conditions for running.

$$q_{1'}(t) := \frac{d}{dt} q_1(t)$$

1st derivative; velocity - fps for running.

$$q_{1''}(t) := \frac{d^2}{dt^2} q_1(t)$$

2nd derivative; acceleration - fpsps for running.

$$q_2(t) := e^{-s \cdot t} \cdot \left(\frac{v_{g2}}{\sqrt{p^2 - s^2}} \right) \cdot \sin\left(\sqrt{p^2 - s^2} \cdot t \right)$$

Exact solution of differential equation using initial conditions for walking.

$$q'_2(t) := \frac{d}{dt} q_2(t)$$

1st derivative; velocity - fps for walking.

$$q''_2(t) := \frac{d^2}{dt^2} q_2(t)$$

2nd derivative; acceleration - fpsps for walking.

$$F_1(t) := \frac{W_b}{g} \cdot q_{1''}(t)$$

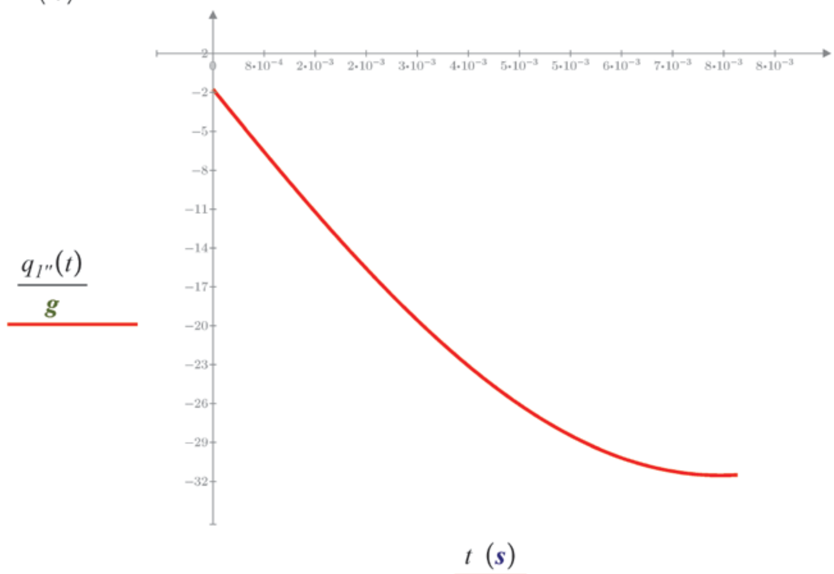
Force on panel for boy running. Where W_b is the applied force on the panel by the arm calculated above.

$$F_2(t) := \frac{W_b}{g} \cdot q''_2(t)$$

Force on panel for boy walking.Where W_b is the applied force on the panel by the arm calculated above.

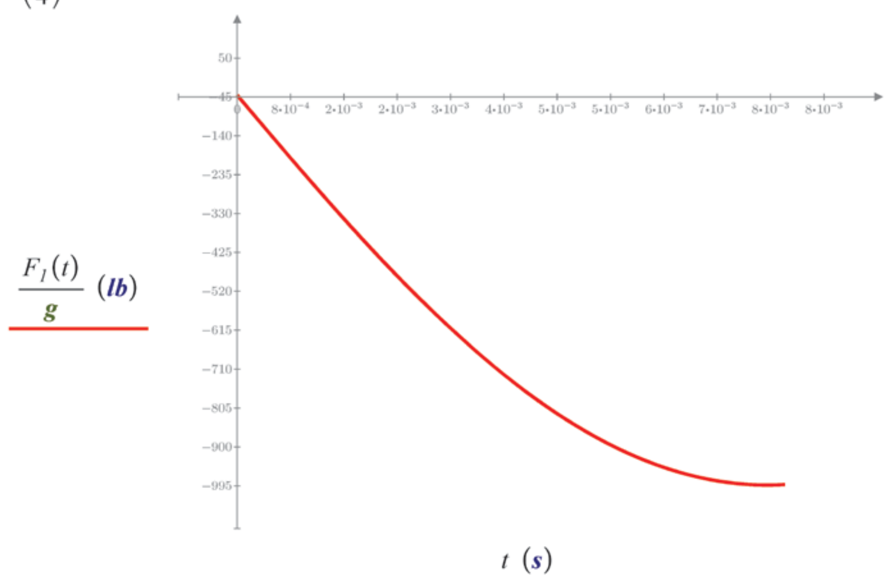
$$q_{1''}\left(\frac{T}{4}\right) = -31\ g$$

Graph 1a. Maximum running g-force on glass panel.



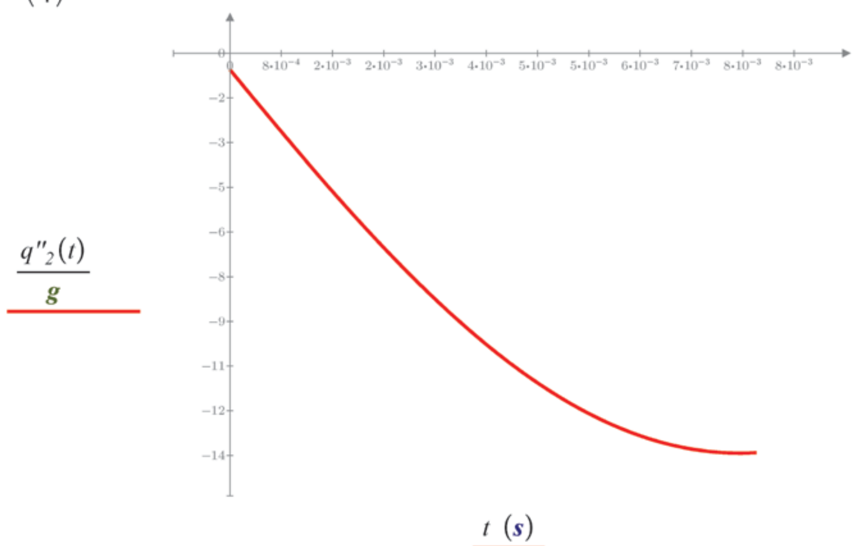
$$F_1\left(\frac{T}{4}\right) = -992\ lbf$$

Graph 1b. Maximum running force on glass panel.



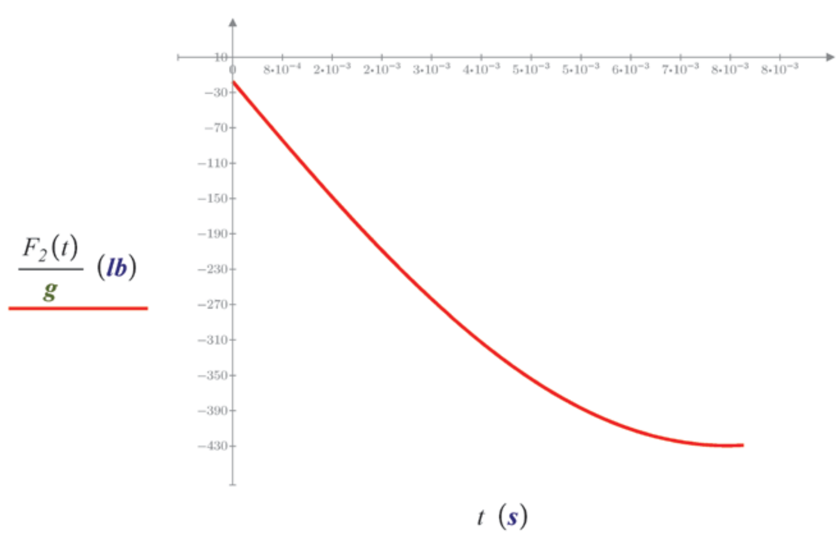
$q''_2\left(\frac{T}{4}\right) = -13.4\ g$

Graph 2a. Maximum walking g-force on glass panel.



$F_2\left(\frac{T}{4}\right) = -429\ \boldsymbol{lbf}$

Graph 2b. Maximum walking force on glass panel.



Calculate the stress on the glass panel using Roark's Stress and Strain Formulas

Reference:

Warren C. Young and Richard G. Budynas, 7th Edition, page 514, Copyright 2002, McGraw-Hill Companies, Inc., Roark's Formulas for Stress and Strain, Uniform over small concentric circle of radius r.o (note definition of r'o), page 514. Figure 9.

See Table 11.4 Formulas for flat plates with straight boundaries and constant thickness. Uniform over small concentric circle of radius r_o where

$$r'_o = \sqrt{1.6\ r_o^{\,2} + t^2} - 0.675\ t \qquad r'_o = r_o \qquad if \qquad r_o \ge 0.5\ t$$

Calculation using Roark's Formula fixed edges.

$a := 81.75 \cdot in$		Long plate dimension.
$b := 45.5 \cdot in$		Short dimension.
$\frac{a}{b} = 1.8$		Ratio a/b for coefficients.
$\beta_1 := 0.068$		Beta coefficient from table.
$t := 0.50005 \cdot in$		Equivalent plate thickness.
$r_o := 2.4 \cdot in$	$r_o = 2.4\ in$	Radius of applied load. (approximate area for hand on glass panel.)
$E := 10000000 \cdot psi$		Young's modulus for glass.
$P := 429 \cdot lbf, 992 \cdot lbf .. 992 \cdot lbf$		Applied loads.
$v := 0.23$		Poisson's ratio.
$\sigma_{mc}(P) := \frac{3 \cdot P}{2 \cdot \pi \cdot t^2} \cdot \left((1+v) \cdot \ln\left(\frac{2 \cdot b}{\pi \cdot r_o}\right) + \beta_1 \right)$		Maximum tensile stress at the center of the panels for fixed edges.

$$\sigma_{mc}(P) = \begin{bmatrix} 2565 \\ 5932 \end{bmatrix} \, psi$$

Maximum stress at the plate center measured 5,932 psi which is greater than the approximate 5,000 psi, the maximum tensile stress of glass. The stress was created with the velocity of the hand/arm at 9.25 feet/second. Therefore, the plate glass would have failed running into the panel.

Maximum stress at the plate center measured 2,565 psi which is less than 5,000 psi, the approximate maximum stress of glass. The stress was created with the velocity of the body at 4 feet/second. Therefore, walking into the glass panel would not have caused the glass to fail.

Calculation using Roark's Formula simply supported edges.
For Uniform over small concentric circle of radius r_o where all edges are simply supported for retangular plate.

$$\beta_2 := 0.927$$

Beta coefficient from Roark table.

$$r'_o := r_o$$

Radius of applied load. (approximate area for hand on glass panel.)

$$\sigma_{max}(P) := \frac{3 \cdot P}{2 \cdot \pi \cdot t^2} \cdot \left((1+v) \cdot \ln\left(\frac{2 \cdot b}{\pi \cdot r'_o} \right) + \beta_2 \right)$$

Maximum tensile stress at the center of the panels for simply supported edges.

$$\sigma_{max}(P) = \begin{bmatrix} 3269 \\ 7559 \end{bmatrix} \, psi$$

Under the same loading conditions, the simply supported edges would undergo higher stresses as compared to the fixed supported edges.

Maximum stress at the plate center measured 7,559 psi which is greater than the approximate 5,000 psi, the maximum tensile stress of glass. The stress was created with the velocity of the hand/arm at 9.25 feet/second. Therefore, the plate glass would have failed running into the panel under the simply supported analysis.

Maximum stress at the plate center measured 3,269 psi which is less than 5,000 psi, the approximate maximum stress of glass. The stress was created with the velocity of the body at 4 feet/second. Therefore, walking into the glass panel would not have caused the glass to fail under the simply supported analysis.