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Forensic Engineering Applications of the G-DaTA*ΔV*™ System of Equations to Real-World Collisions

By Jerry S. Ogden, Ph.D., P.E. (NAFE 561F)

Abstract

Analysis of vehicle deformation from impacts largely relies upon A and B stiffness coefficients for vehicle structures in order to approximate the velocity change and accelerations produced by an impact. While frontal impact stiffness factors for passenger vehicles, light trucks, vans, and sport utility vehicles are relatively prevalent for modern vehicles, stiffness factors for rear and side structures, as well as heavy vehicles, buses, recreational vehicles, trailers, motorcycles, and even objects, are essentially non-existent.

*This paper presents the application of the Generalized Deformation and Total Velocity Change Analysis to real-world collision events (**G-DaTAΔV™ System of Equations**) as developed by this author. The focus of this paper addresses the relative precision and accuracy of the G-DaTAΔV™ System of Equations for determining the total velocity change for oblique and/or offset vehicle-to-vehicle collisions involving light trucks and sport utility vehicles, which are largely under-represented with modern vehicle A and B stiffness values for side and rear surfaces. The previous paper presented by this author to the Academy addressed the relative accuracy and precision of the G-DaTAΔV™ System of Equations as they relate to a first validation using the RICSAC-staged collision database[1]. As a secondary and more comprehensive validation process, the G-DaTAΔV™ System of Equations will be applied to real-world collision data obtained through the National Automotive Sampling System (NASS), which provides the National Highway Traffic Safety Administration (NHTSA) with a comprehensive compilation of real-world collision events representing a broad-based collection of collision configurations from across the country. This data represents a reusable source of information that was collected using standardized field techniques implemented by NASS-trained field technicians. Through using a "core set of crash data components," NASS has demonstrated its utility and applicability to a vast array of statistical and analytical studies regarding traffic safety and vehicle collision dynamics[2].*

Keywords

Forensic engineering, force deflection, damage analysis, missing vehicle stiffness, total velocity change, crush energy, G-DaTA*ΔV*™ System of Equations

Background

The G-DaTA*ΔV*™ System of Equations, as presented in this Journal previously[1], provides the following significant advancements and/or enhancements to modern vehicle deformation-based analysis methodologies:

- Eliminates the dependence upon multiple structural stiffness coefficients for permanent vehicle structural deformation analysis, regardless of the impacted surface and vehicle type involved.
- Account for oblique and off-set collisions that result in principal direction of force that do not pass through the mass centers of vehicles and produce rotation.
- Account for inter-vehicular friction due to the colliding surfaces of vehicles sliding during the approach velocity change of an impact.
- Account for external tire-ground forces during the approach velocity change of an impact.
- Define the total velocity change resulting from any collision event, which considers the velocity

Jerry S. Ogden, Ph.D., P.E., P.O. Box 621519, Littleton, Colorado 80162-1519; (303) 795-1515; jogden@OEC4N6.com

change resulting from linear and rotational momentum (conservative forces) as well as the contributions due to inter-vehicular friction and tire/ground forces (non-conservative forces).

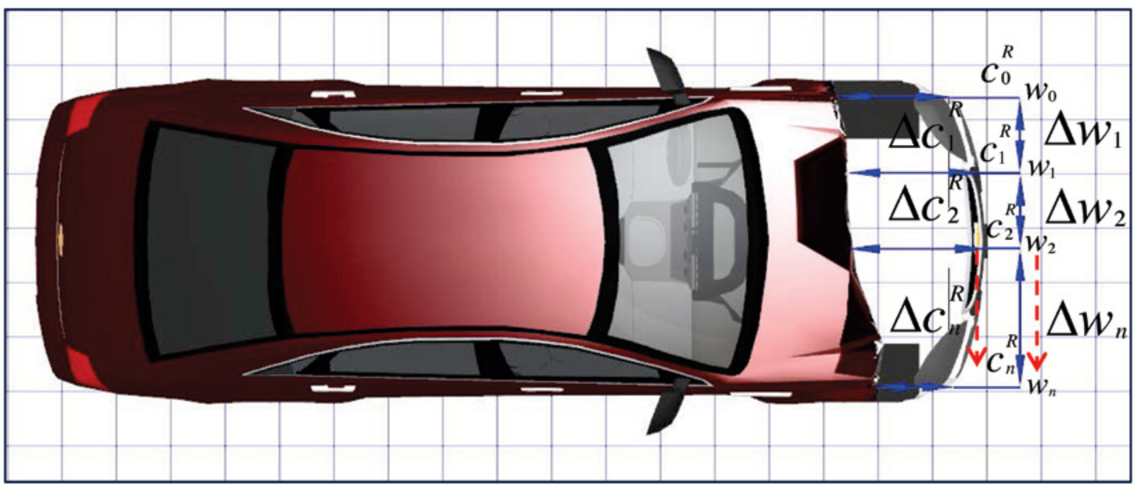
The G-DaTAΔV™ System of Equations showed excellent correlation to the RICSAC test data with an $R^2 = 0.989$ for piecewise damage profile analysis and an $R^2 = 0.991$ for the weighted average damage profile analysis. The $\chi^2 = 1.06$ for the piecewise and $\chi^2 = 1.08$ for the weighted average damage profile analysis methodologies ($\alpha=0.99$, $n=23$), which indicates the difference between the total velocity changes for the RICSAC tests and calculated values using either G-DaTAΔV™ System of Equations methodological approach, is not statistically significant when applied to the RICSAC-staged collision testing[1].

G-DaTAΔV™ Analysis Procedure

As mentioned, the application of the G-DaTAΔV™ System of Equations was outlined previously by this author[1]. In order to maintain uniformity between studies, the equation numbers from reference 1 will be used in this paper for the equations of the G-DaTAΔV™ System of Equations. Analysis using this approach starts with the documentation of vehicle deformation profiles for each vehicle into the form demonstrated in **Figure 1**.

After tabulating the deformation profiles for the numerical analysis, the following general analytical steps provide the *total velocity change* for two colliding vehicles:

1) Obtain vehicle weights, dimensions and determine inertial properties (Equation 10 from reference 1)



Description of point measured	Distance from Left Front (0)	Distance to undamaged profile
Left front corner	w0: +0 cm (0 inches)	c0: +81 cm (32 inches)
Left front frame rail	w1: +46 cm (18 inches)	c1: +81 cm (32 inches)
Center bumper reinforcement bar	w2: +92 cm (36 inches)	c2: +71 cm (28 inches)
C3…C(n-1)	w3…w(n-1)	c3…c(n-1)
Right front corner	w(n): +183 cm (72 inches)	c(n): +51 cm (20 inches)

So that,

$$\Delta c_1^R = \frac{c_0^R + c_1^R}{2} = \frac{(81cm + 81cm)}{2} = 81cm \text{ (32 inches)}$$

$$\Delta c_2^R = \frac{c_1^R + c_2^R}{2} = \frac{(81cm + 71cm)}{2} = 76cm \text{ (30 inches), and so forth.}$$

And,

$$\Delta w_1 = w_1 - 0_0 = (46cm - 0cm) = 46cm \text{ (18 inches)}$$

$$\Delta w_2 = w_2 - w_1 = (92cm - 46cm) = 46cm \text{ (18 inches), and so forth.}$$

Figure 1
Measured damage dimensions.

$$I_{zz} = \frac{m_{curb}}{K_G} \cdot \left(L^2 + b^2\right) \cdot \left(K_M \cdot \left(\frac{m_{loaded} - m_{curb}}{m_{loaded}}\right)\right) \quad 1$$

Where, I_{zz} = yaw moment of inertia (about *z-axis*)
m_{curb} = curb mass of vehicle (unloaded)
m_{loaded} = loaded mass of vehicle (curb plus occupants and cargo)
L = total length of vehicle
b = maximum width of vehicle
K_G = geometric empirically determined constant (see **Figure 2**)
K_M = geometric empirically determined constant (see **Figure 2**)

Vehicle type	K_G	K_M	R^2
All combined	13.1	0.696	0.85
Passenger car	13.8	0.769	0.86
Light truck	13.4	0.750	0.92
SUV	12.2	0.656	0.76
Light van	12.3	0.642	0.90

Figure 2
Yaw moment of inertia empirical constants[3].

2) Determine the PDOF acting upon each vehicle, which will be directly opposite in direction when the vehicles are placed together at maximum engagement; **Figure 3** as adapted from **Figure 5** from reference 1.

3) Obtain vehicle A/B stiffness values for the selected vehicle in determining the generalized force acting equal and opposite between the colliding vehicles (Equation 11 from reference 1) based upon the following hierarchy:

$$F^{Gen} = \sum_{j=0}^{n}\left(A_i + B_i \cdot \Delta c_j^R\right) \cdot \frac{\Delta w_j}{\cos(\Pi_i^{PDOF})} \quad 2$$

Where, A_i and B_i = unique structural stiffness values for the impacted surface of the selected vehicle of known A/B values.
Δc_j^R = the residual deformation, or "crush," of the j[th] deformation measured on the selected vehicle perpendicular to the damaged surface from its undamaged dimensions.
Δw_j = width of the j[th] deformation, measured parallel to the damaged surface of the selected vehicle.
Π_i^{PDOF} = angle of the PDOF acting upon the selected vehicle.

a) If both colliding vehicles have frontal stiffness values available, choose the A/B stiffness value for the vehicle with the greatest extent of measured damage (damage width and depth profile).
b) Frontal A/B stiffness for vehicle with frontal impact damage for oblique side, broadside, and rear-end impact configurations.
c) A/B stiffness by vehicle struck surface (front, rear, or side) if only one vehicle has an impact surface that is supported by test data regardless of impact configuration.
d) If neither vehicle impact surface is supported, use a range of A/B stiffness factors for similar vehicles to establish a higher and lower bounding for the analysis.

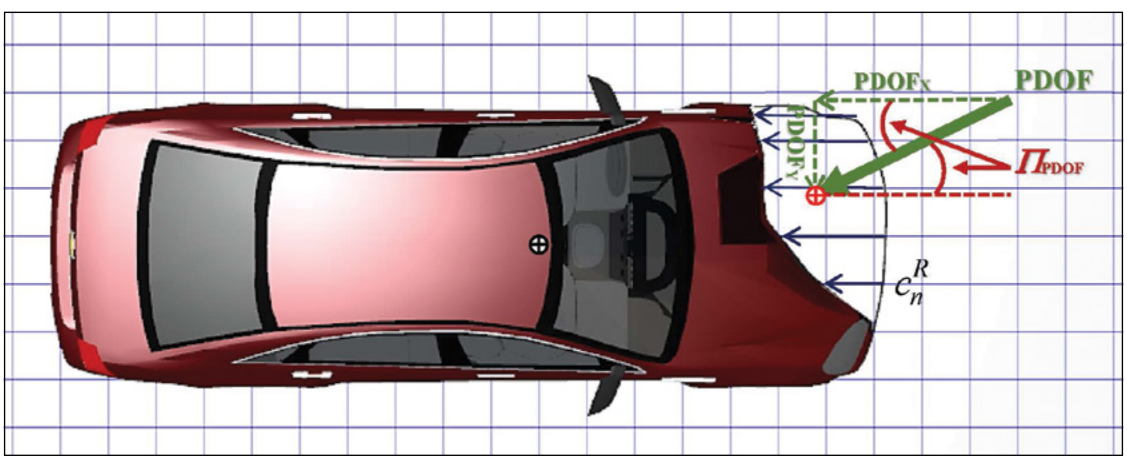


Figure 3
Oblique impact PDOF acting at damage centroid.

4) Determine the work due to the non-conservative inter-vehicular friction forces (Equations 17 and 18 from reference 1).

$$E_1^{friction} = m_1 \cdot g \cdot \mu_k \cdot \left(\frac{m_2}{m_1 + m_2} \right) \cdot \Delta w_{scrape} \quad (3)$$

$$E_2^{friction} = m_2 \cdot g \cdot \mu_k \cdot \left(\frac{m_1}{m_1 + m_2} \right) \cdot \Delta w_{scrape} \quad (4)$$

Where, m_1 and m_2 = masses of colliding vehicles 1 and 2, respectively (mass units)
μ_k = inter-vehicular friction due to surface scraping
Δw_{scrape} = difference in deformation contact widths (scrape distance)

5) Determine the weighted average deformation depth for the vehicle that is <u>not supported</u> by A/B stiffness data or where A/B stiffness data was not used (Equation 27 from reference 1).

$$\overline{C}^{unknown} = \frac{\sum_{j=1}^{n} \Delta w_j \cdot \Delta c_j}{\sum_{j=1}^{n} \Delta w_j} \quad (5)$$

Δw_j and Δc_j from **Figure 1**

6) Determine the *generalized work* to produce compression of the vehicle structures in the form of permanent deformation (Equations 19 and 26 from reference 1).

7) Determine the time period to reach maximum impulse, which is <u>not</u> the total time of the impact to reach maximum velocity change, but the time in which the peak force is applied during the impact (Equation 24 from reference 1).

$$\Delta t^{Gen} = \sqrt{\left(\frac{2 \cdot m_1 \cdot \gamma_1 \cdot m_2 \gamma_2}{(m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2)} \right) \cdot \frac{\left(E_1^{Gen} + E_2^{Gen} \right)}{\left(F^{Gen} \right)^2}} \quad (8)$$

Where,

$$\gamma_1 = \left(\frac{I1_{zz}}{I1_{zz} + m_1 \cdot h_1^2} \right) \quad \text{and,} \quad \gamma_2 = \left(\frac{I2_{zz}}{I2_{zz} + m_2 \cdot h_2^2} \right)$$

And, h_1 and h_2 moment arms of PDOF from mass centers of vehicles 1 and 2, respectively.

8) Determine the roadway friction (μ) and equivalent braking efficiency (n) for the vehicle whose tires act against the direction of impact force application (struck vehicle).

9) Determine an appropriate coefficient of restitution for the impact. The following are general rules for determining appropriate coefficients of restitution:
   a) Minor impacts with minor damage will have higher restitution values[4,5].
   b) Even with extensive permanent damage profiles, ranging restitution between 0 and 0.1 may provide a greater confidence interval in the analysis results[1,11].
   c) When the impact involves an axle and/or wheel of a struck vehicle in an oblique side or broadside impact, restitution will range from 0.2 to 0.4 to account for the hardened zone of the axle and/or the "bounce" effect of impacting an inflated tire[1].

10) Determine the *total velocity change* for the vehicles produced by the impact event (Equations 22 and 23 of reference 1).

$$E_1^{Gen} = E_1^{Oblique} + E_1^{friction}$$
$$= \sum_{j=0}^{n} \left(A_1 \cdot \Delta c1_j^R + \frac{B_1 \cdot \left(\Delta c1_j^R \right)^2}{2} + \frac{A_1^2}{2 \cdot B_1} \right) \cdot \Delta w1_j \cdot \left(1 + \tan^2 \left(\Pi_{PDOF1} \right) \right) + m_1 \cdot g \cdot \mu_k \cdot \left(\frac{m_2}{m_1 + m_2} \right) \cdot w_1 \quad (6)$$

$$\left(E_{work}^{Gen} \right)^{unknown} = \left(F^{Gen} \right)^{known} \cdot \left(\overline{C}^{unknown} \right) \cdot \left(1 + \tan^2 (\Pi^{unknown}) \right) \quad (7)$$

$$\Delta V_1^{Gen} = (1+e)\sqrt{\frac{2\cdot m_2\cdot\gamma_2\cdot\left(E_1^{Gen}+E_2^{Gen}\right)}{m_1\cdot\gamma_1\cdot\left(m_1\cdot\gamma_1+m_2\cdot\gamma_2\right)}}+\frac{m_2\cdot(g\cdot\mu\cdot n)\cdot\Delta t}{m_1} \quad 9$$

$$\Delta V_2^{Gen} = (1+e)\sqrt{\frac{2\cdot m_1\cdot\gamma_1\cdot\left(E_1^{Gen}+E_2^{Gen}\right)}{m_2\cdot\gamma_2\cdot\left(m_1\cdot\gamma_1+m_2\cdot\gamma_2\right)}}-(g\cdot\mu\cdot n)\cdot\Delta t \quad 10$$

Outside of accurate deformation profile measurements, Step 3 is perhaps the most crucial in the application of the G-DaTAΔV™ System of Equations. The determination of the *generalized force* of the impact is completed for only one vehicle, not for both, since by Newton's third law the *generalized force* acting upon both vehicles is equal in magnitude but opposite in direction of application. If reliable stiffness data is available for both colliding vehicles and for the appropriate colliding surfaces (front, rear, or side), then the determination of the *total velocity change* for each vehicle can be calculated by applying the G-DaTAΔV™ System of Equations twice and comparing results as a useful crosscheck or for providing a reasonable confidence interval for the analysis[1].

The equations presented in this section comprise the G-DaTAΔV™ System of Equations as they relate to the determination of the *total velocity change* of a vehicle-to-vehicle or vehicle-to-object collision event.

Application of G-DaTAΔV™ to NASS Real-World Collisions

Twenty-five collisions were selected from the NASS Crashworthiness Data System (CDS) Case Viewer from the 2004 to 2013 approved data set, which met the following specific criteria for consideration:

- Two-vehicle collisions involving at least one light truck/van or one SUV category vehicle, with a preference to collisions involving only these category vehicles.

- At least one vehicle must have a complete Event Data Recorder (EDR) imaged report using the Bosch Crash Data Retrieval Tool (Bosch CDR Tool) without evidence of significant data clipping or incomplete data records due to power interruptions or system failures, with preference upon collisions involving both vehicles having a CDR report.

- Both colliding vehicles have complete measured damage profiles consistent with photographs documenting the post-collision condition of each vehicle.

- One vehicle must have Neptune Engineering NEI[6] database reported A and B structural stiffness coefficient values specific to the vehicle and impacted surface or applicable for sister model year runs or corporate manufacturer clones.

The NASS database provides the year, make, and model of each colliding vehicle and the standard curb weight from various sources, some of which are non-standard sources, as well as the occupant and cargo load at the time of impact (when known). However, the vehicles are not weighed by the field investigators, and the mass center or weight distribution is not determined for any vehicles. Therefore, in order to replicate real-world analysis procedures, which would likely be followed for individual collision reconstructions, the standard curb weights and distributions were determined using an industry resource[7] and while adding occupant and cargo loads. Additionally, the NASS database does not provide a measured drag factor for the individual roadway surfaces of the reported collisions. Accordingly, a uniform approximation of a dry roadway drag factor of m = 0.80 was used as the baseline roadway friction for each analysis. The structural stiffness data for one of the colliding vehicles was obtained through the NEI database. The following additional variables necessary for the G-DaTAΔV™ System of Equations analysis were available from the NASS database for each collision as follows:

- Vehicle collision deformation width and depth profiles (measured in SI units).

- Diagrams at impact, post-collision trajectories and tire marks, and vehicle final rest locations.

- Contact with wheel/tire hard zones for restitution considerations provided through vehicle photographic evidence, evidence (when appropriate).

- EDR output images using the Bosch CDR Tool for at least one of the vehicles, having both longitudinal and lateral total velocity change recordings.

The vehicles were positioned together at maximum engagement with the PDOF passing through the damage centroids as discussed in reference 1 and shown in **Figure 3**. The PDOF acting upon each vehicle was determined from the total velocity change vectors determined from the velocity recordings image from the vehicle EDRs. The moments of inertia for the vehicles were determined using Equation 10. Since all dimensions in the NASS-reported collisions were reported in SI units, the moment arm for the offset and oblique impacts were measured using the Faro Reality CAD program to within 0.1m[8]. Damage width and depth dimensions were used as reported for each collision, which were measured to the nearest centimeter. All data recorded within the NASS reports of each real-world collision event was used as reported with no interpretation or modifications. PTC® MathCAD Prime® 3.0 was used for the calculations, which completes all unit conversions internally so that the potential for unit conversion errors were eliminated[9].

The purpose of the final evaluation using the NASS-reported real-world collision data is to determine the accuracy and precision of the G-DaTAΔV™ System of Equations developed in this study as they relate to the data typically available or obtained during a real-world collision investigation. The capstone contribution of this study involves the incorporation of all of the contributions to the total velocity change produced by an oblique, offset, and non-central impact applied to the NASS real-world collisions involving SUVs and light trucks, which have minimal structural stiffness data for side and rear structures.

NASS G-DaTAΔV™ System of Equations Analysis Results

Due to NASS data collection practices and/or the lack of SUV and light trucks involved with downloadable EDRs capable of recording acceleration collision pulses, the year range of NASS-reported collision data that met the established criteria of this study was limited to the collision years of 2010 to 2013. Some NASS-reported collisions involved only one vehicle with a complete longitudinal and lateral Bosch CDR Tool report, while the other involved vehicle was limited to an earlier generation EDR that provided only one direction (lateral or longitudinal) of EDR recording. The condition when Bosch CDR Tool records contained only one direction of velocity change data (either longitudinal or lateral) is easily resolved by the following steps:

- Determine the principal direction of force (PDOF) acting upon the vehicle with a complete longitudinal and lateral Bosch CDR Tool report of the collision total velocity change vector.
- Position the vehicles together using a collision diagram as shown in **Figure 3**.
- Use trigonometric identities in determining the total velocity change for the collision of the vehicle having only a single reported velocity change vector and the PDOF acting upon the vehicle determined from the collision diagram.

Figure 4 summarizes the raw calculation results, and **Figure 5** provides the statistical analysis while determining the vehicle total velocity change of the NASS data utilizing the G-DaTAΔV™ System of Equations**.** The data is also plotted for linearity in **Figure 6** regarding the piecewise damage profile analysis and **Figure 7** for the weighted average damage profile analysis methodological approaches.

NASS #	ΔV CDR mph	PIECEWISE ΔVtotal mph	Difference (Calc-Test) mph	Percent Difference (Calc-Test)	WEIGHTED AVERAGE ΔVtotal mph	Difference (Calc-Test) mph	Percent Difference (Calc-Test)	Collision Description
2010-08-037 Veh 1	19.1	19.7	0.5	2.8%	19.0	-0.2	-0.9%	09 Toyota Tacoma PU into LR of 09 Pontiac G6 1 CDR
2010-12-154 Veh 1	19.3	20.5	1.3	6.5%	20.3	1.0	5.4%	09 Pontiac G6 frontal 1 CDR
2010-12-154 Veh 2	7.7	8.6	0.8	10.7%	8.5	0.7	9.6%	2010 Ford F150 RT side 2 CDR
2011-04-127 Veh 1	13.6	13.6	0.0	-0.4%	13.5	-0.1	-0.6%	05 Equinox strikes side of 02 Explorer 1 CDR
2011-08-107 Veh 1	10.2	10.4	0.1	1.2%	10.5	0.2	2.3%	01 Buick LeSabre frontal 1 CDR
2011-08-107 Veh 2	9.1	8.9	-0.2	-2.4%	9.0	-0.1	-1.3%	08 Chrysler Aspen side 2 CDR
2011-08-112 Veh 1	10.5	10.4	-0.1	-1.1%	10.1	-0.4	-3.7%	05 Chevrolet Equinox frontal 1 CDR
2011-08-112 Veh 2	11.8	11.5	-0.3	-2.6%	11.2	-0.6	-5.2%	06 Chevrolet Impala LF side 2 CDR
2011-09-075 Veh 1	13.1	14.5	1.4	10.7%	14.4	1.3	10.2%	10 Buick Lacrosse into LR side 08 Honda Ridgeline 1 CDR
2011-09-091 Veh 1	31.0	31.8	0.8	2.4%	31.2	0.2	0.8%	03 GMC 2500 into LF side 07 Acura MDX CDR 1
2011-11-085 Veh 2	13.9	14.1	0.2	1.5%	13.8	-0.1	-0.8%	04 Hyundai Sonata into LF side of 11 Ford Escape CDR1
2011-12-049 Veh 1	20.7	19.5	-1.2	-5.9%	18.9	-1.8	-8.7%	07 Chev Equinox into RF of 07 GMC 1500 CDR 1
2011-12-046 Veh 2	13.1	13.0	-0.1	-0.7%	12.7	-0.5	-3.7%	07 GMC 1500 RF struck by 07 Chev Equinox CDR 2
2011-12-189 Veh 1	10.1	10.6	0.5	5.1%	10.5	0.4	4.1%	11 Chev Equinox LF into RT side 06 Chev Impala CDR 1
2011-12-189 Veh 2	11.8	11.2	-0.6	-4.9%	11.1	-0.7	-5.8%	06 Chev Impala RT side struck LF 11 Chev Equinox CDR 2
2012-08-064 Veh 1	14.7	14.5	-0.2	-1.4%	14.2	-0.6	-3.9%	09 Cadillac Escalade LF into LF corner 98 Honda Accord CDR
2012-08-080 Veh 1	7.6	8.0	0.4	5.5%	8.1	0.5	6.2%	10 GMC Yukon LF corner sideswipe LT side 12 Toyota 4Run CDR1
2012-08-080 Veh 2	7.7	8.3	0.6	7.8%	8.3	0.6	8.4%	12 Toy 4Run LT sideswipe by LF corner 10 GMC Yukon CDR 2
2012-12-016 Veh 1	13.2	13.6	0.5	3.5%	13.4	0.2	1.7%	08 Cadillac CTS into side of 08 Chevrolet Trailblazer CDR 1
2012-12-016 Veh 2	13.8	15.3	1.5	10.6%	15.0	1.2	8.7%	08 Chevrolet Trailblazer side struck by 08 Cadillac CTS front CDR 2
2012-41-024 Veh 1	32.8	34.2	1.4	4.2%	32.8	0.0	-0.1%	05 Toyota Camry LF into 2010 Toyota Tundra RF CDR 1
2012-41-024 Veh 2	22.2	20.5	-1.6	-7.4%	19.7	-2.5	-11.3%	2010 Toyota Tundra RF into 05 Toyota Camry LF CDR 2
2012-43-014 Veh 1	30.8	33.5	2.7	8.7%	31.8	1.0	3.4%	2011 Jeep Liberty into side of 2011 Ford F250 CDR 1
2012-43-014 Veh 2	19.2	19.9	0.7	3.9%	18.9	-0.2	-1.3%	2011 Ford F250 struck on RT by 2011 Jeep Liberty CDR 2
2012-43-026 Veh 1	22.7	20.6	-2.0	-9.0%	20.4	-2.3	-10.0%	2009 Lexus RX350 strikes LF of 2005 Toyota Camry CDR 1
2012-43-026 Veh 2	24.1	26.0	1.9	8.0%	25.7	1.6	6.8%	2005 Toyota Camry LF struck by front of 2009 Lexus RX350 CDR 2
2012-43-106 Veh 1	25.4	27.5	2.1	8.4%	26.9	1.5	6.0%	2001 Lincoln Navigator strikes LT 2011 Dodge Durango CDR 1
2012-43-106 Veh 2	20.3	18.8	-1.5	-7.6%	18.3	-2.0	-9.6%	2011 Dodge Durango LT struck by 2001 Lincoln Navigator CDR 2
2012-48-106 Veh 1	13.0	13.4	0.4	3.0%	13.2	0.2	1.3%	2007 Toyota FJ Cruiser into LT 2007 Toyota RAV4 CDR 1
2012-48-106 Veh 2	13.3	13.3	0.0	0.1%	13.1	-0.2	-1.5%	2007 Toyota RAV4 LT struck by 2007 Toyota FJ Cruiser CDR 2
2013-12-059 Veh 1	8.7	9.9	1.3	14.4%	9.5	0.9	10.0%	2006 Chevrolet K1500 into LT 2005 Chevrolet Malibu CDR1
2013-12-059 Veh 2	17.1	17.3	0.3	1.7%	18.6	1.5	9.0%	2005 Chevrolet Malibu LT struck by 2006 Chevrolet K1500 CDR 2
2013-12-106 Veh 1	25.9	27.4	1.6	6.1%	27.1	1.3	5.0%	2012 Chevrolet Equinox into RT 2008 GMC C2500 w/trailer CDR 1
2013-12-106 Veh 2	10.3	9.7	-0.6	-5.7%	9.6	-0.7	-6.7%	2008 GMC C2500 w/trailer RT struck by 2012 Chevrolet Equinox CDR 2
2013-12-112 Veh 1	26.9	27.0	0.2	0.6%	26.3	-0.6	-2.1%	2004 Chevrolet Venture into RT 2012 Chevrolet Equinox CDR1
2013-12-112 Veh 2	17.9	16.8	-1.1	-6.2%	16.4	-1.6	-8.7%	2012 Chevrolet Equinox RT struck by 2004 Chevrolet Venture CDR 2
2013-43-152 Veh 1	14.0	13.1	-0.8	-6.0%	12.5	-1.5	-10.4%	1997 Chevrolet C1500 into LT 2011 Ford Ranger CDR 1
2013-43-152 Veh 2	14.2	13.4	-0.9	-6.2%	12.7	-1.5	-10.6%	2011 Ford Ranter LT struck by 1997 Chevrolet C1500 CDR 2
2013-76-094 Veh 1	19.1	16.7	-2.5	-12.9%	16.7	-2.4	-12.8%	2010 Dodge Journey into RT 2007 Pontiac Torrent CDR 1
2013-76-094 Veh 2	15.2	15.8	0.5	3.5%	15.8	0.5	3.6%	2007 Pointiac Torrent RT struck by 2010 Dodge Journey CDR 2
2013-76-165 Veh 1	21.8	22.6	0.8	3.8%	22.3	0.5	2.4%	2013 Ford F150 into LT side of 2013 Ford F150 CDR 1
2013-76-165 Veh 2	15.8	16.8	1.1	6.7%	16.6	0.8	5.2%	2013 Ford F150 LT struck by 2013 Ford F150 CDR 2
2013-79-139 Veh 1	32.0	32.4	0.4	1.4%	30.3	-1.7	-5.2%	2004 Toyota Prius head-on offset with 2007 Toyota Highlander CDR 1
2013-79-139 Veh 2	27.6	29.7	2.1	7.6%	27.7	0.1	0.5%	2007 Toyota Highlander head-on offset with 2004 Toyota Prius CDR 2
Absolute Averages	17.3	17.6	0.9	5.2%	17.2	0.9	5.4%	

Figure 4
NASS-reported Bosch CDR Tool data versus G-DaTAΔV™ analysis.

Piecewise Matching Method			Weighted Average Method		
χ2 Test of fit	2.92	standard deviation of error	χ2 Test of fit	2.98	standard deviation of error
χ2 critical => (α = 0.99)	24.43 (n=43)	±1.1 mph	χ2 critical => (α = 0.99)	24.43 (n=43)	±1.1 mph
R^2 =>	0.979	±6.3%	R^2 =>	0.975	±6.7%
T-test => (p=0.9129)	0.11		T-test => (p=0.6478)	0.46	
critical=> (α = 0.05)	2.017		critical=> (α = 0.05)	2.017	
F-test =>	0.75		F-test =>	0.99	
critical=> (α = 0.05)	3.209		critical=> (α = 0.05)	3.209	

Figure 5
Summary of statistics.

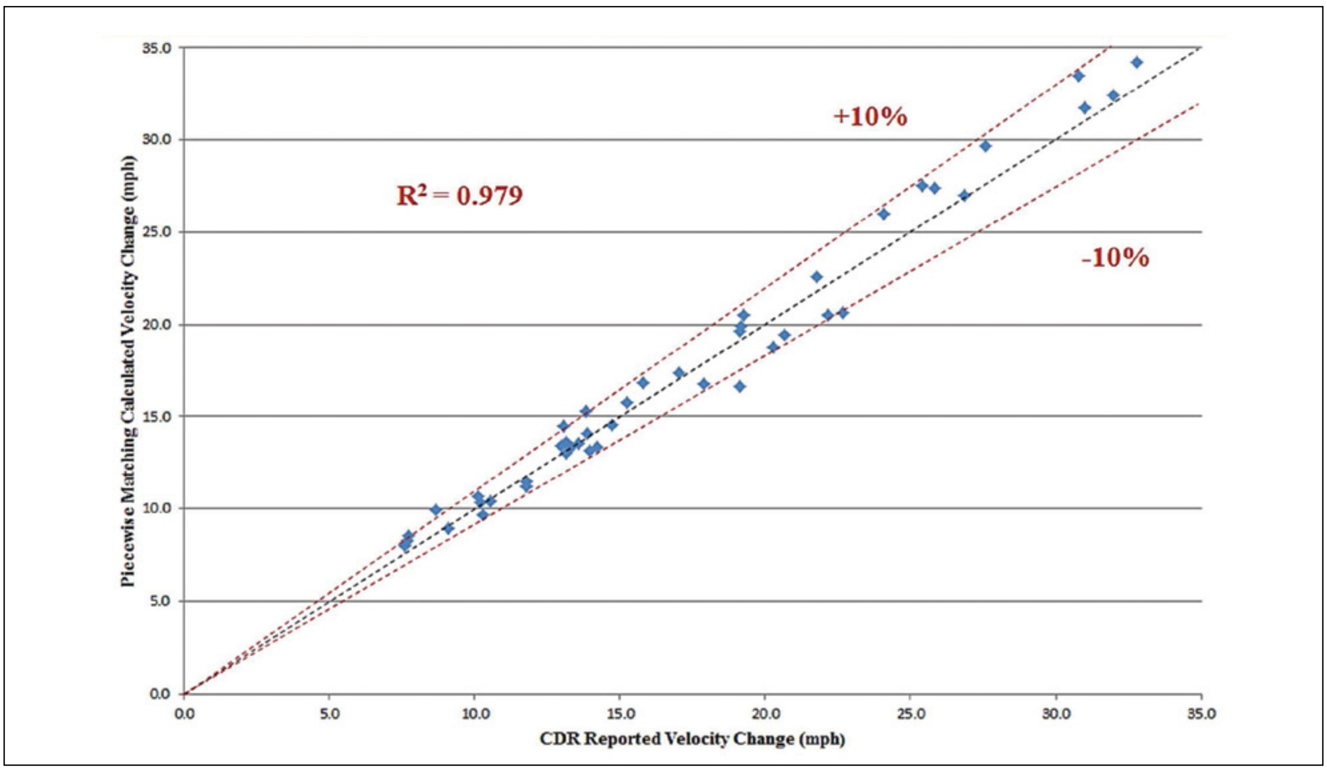


Figure 6
G-DaTA*ΔV*™ piecewise damage match versus NASS Bosch CDR data.

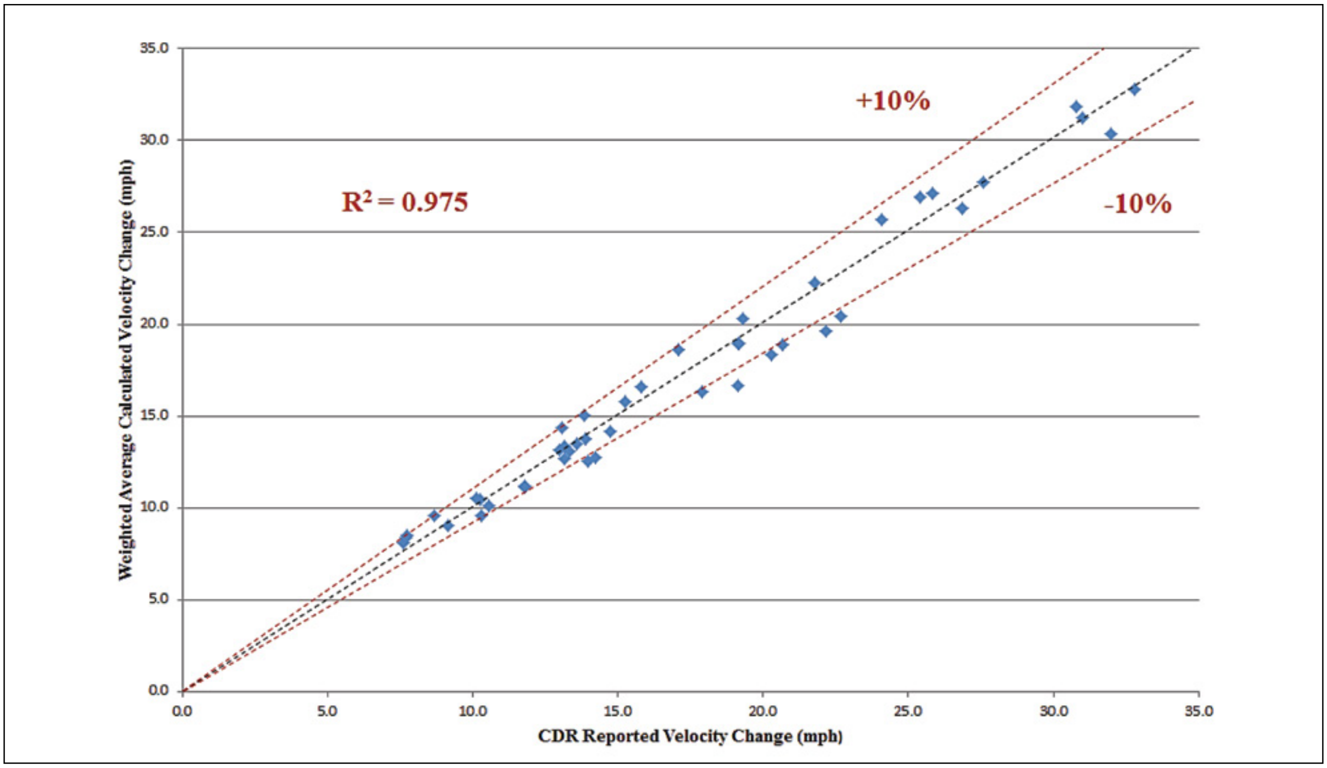


Figure 7
G-DaTA*ΔV*™ weighted average damage versus NASS Bosch CDR data.

The G-DaTA*ΔV*™ System of Equations showed excellent correlation to the NASS total velocity change Bosch CDR Tool reported data with an $R^2 = 0.979$ for piecewise damage profile analysis and an $R^2 = 0.975$ for the weighted average damage profile analysis. The $\chi^2 = 2.92$ for the piecewise and $\chi^2 = 2.98$ for the weighted average damage profile analysis methodologies ($\alpha=0.99$, $n=43$) indicate that the difference between the total velocity changes for the NASS Bosch CDR Tool reported real-world collisions and calculated values using either G-DaTA*ΔV*™ System of Equations methodological approach is not statistically significant.

The overall precision of the results varies by ±6.3% (±1.1 mph) for the piecewise method and ±6.7% (±1.1 mph) for the weighted average damage profile methods for errors within one standard deviation of the mean. With respect to the piecewise damage profile method, the greatest percentage differences between the calculated and NASS real-world collision results varied between -12.89% (-2.46 mph difference) for vehicle 1 of NASS 2013-76-094 to +14.4% (1.25 mph difference) for vehicle 1 of NASS 2013-12-059. The precision utilizing the weighted average damage profile method improved to -12.81% (-2.45 mph difference) for vehicle 1 of NASS 2013-76-094 to +10.0% (0.87 mph difference) for vehicle 1 of NASS 2013-12-059. The fact that the outliers for both methods involved the same vehicles from the same reported collisions could be random, but is probably due to a systematic error in the data reported within the particular NASS files.

The relative high degree of correlation between the NASS-reported total velocity changes (from the vehicle EDR data as imaged within their respective Bosch CDR Tool reports), as compared to the results when utilizing the G-DaTA*ΔV*™ System of Equations, indicates that the suite of equations produced reasonable precision and accuracy for determining the *total velocity change* resulting from these real-world collision events. Additionally, the evaluation results from utilizing the G-DaTA*ΔV*™ System of Equations indicates the NASS training of investigators regarding vehicle deformation documentation appears adequate for reducing random and/or systematic errors between investigators.

Application Examples of the G-DaTA*ΔV*™ System of Equations

The following example application of the G-DaTA*ΔV*™ System of Equations is from NASS-reported collision 2010-08-037 involving a large amount of inter-vehicular friction due to the oblique-offset impact configuration of the collision event between a 2009 Toyota Tacoma and a 2009 Pontiac

G6[10]. The following **Figures 8** and **9** from the NASS report detail the damages to the vehicles from the impact. Measurements of damage profiles as well as CDR downloads were also part of the NASS report.

In accordance with steps 1 through 10 of the analysis procedures established for the application of the G-DaTAΔV™ System of Equations, the vehicles were placed together at maximum engagement for the determination of the *total velocity change* of each vehicle resulting from the impact. The collision diagram shown as **Figure 10** and subsequent MathCAD Prime 3.0 worksheets detail the analysis approach, as well as the mathematical results as compared to the *total velocity change* levels imaged in the vehicle CDR report.

Figure 8
Pontiac G6 damage diagram and sample photographs.

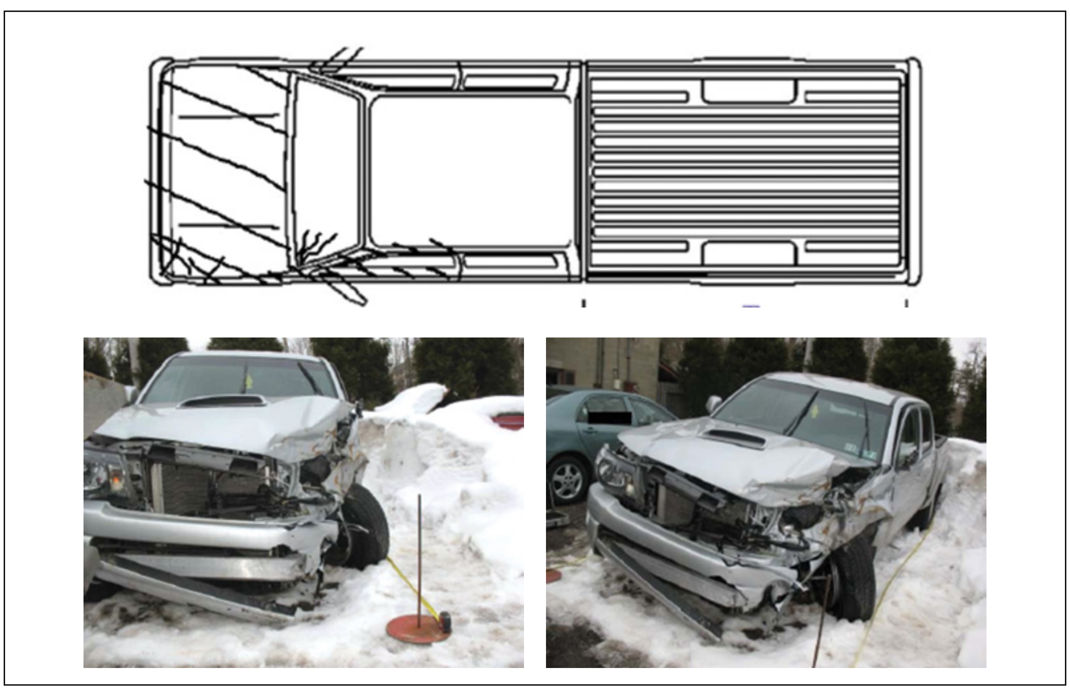


Figure 9
Toyota Tacoma damage diagram and sample photographs.

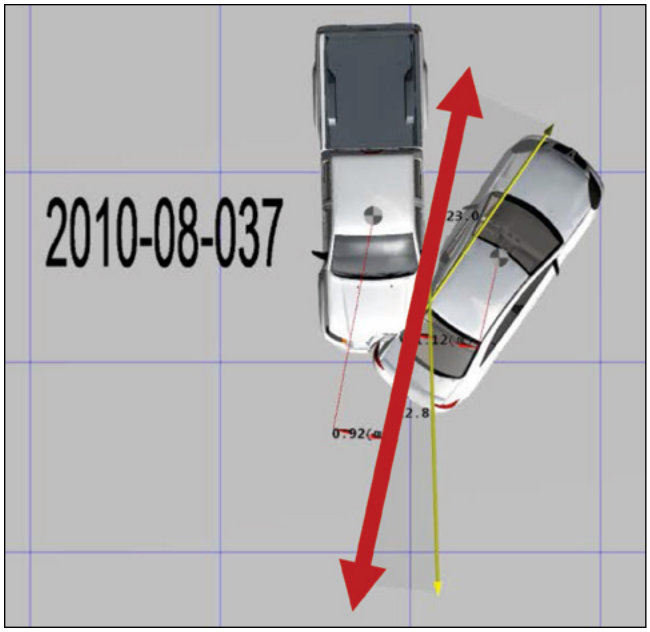


Figure 10
Maximum engagement diagram with moment arms and
PDOF of applied force.

$$C1 := \begin{bmatrix} 25 \\ 18 \\ 16 \\ 14 \\ 12 \\ 10 \\ 0.0 \end{bmatrix} \cdot cm \qquad W1 := \begin{bmatrix} 0 \\ 17 \\ 34 \\ 51 \\ 68 \\ 85 \\ 102 \end{bmatrix} \cdot cm$$

$$m_1 := \frac{(4259) \cdot lb}{g} = 132.4\ \frac{lb \cdot s^2}{ft}$$ 2009 Toyota Tacoma

$$I1_{zz} := 2313.50 \cdot lb \cdot ft \cdot sec^2$$ From 4N6Xpert Autostats

$$A1 := 296 \cdot \frac{lb}{in} \qquad B1 := 100 \cdot \frac{lb}{in^2}$$ Neptune 04-05 Tacoma frontal

$$\Pi_{PDOF1} := (-12.8) \cdot deg \qquad h_1 := 0.9 \cdot m \qquad \gamma_1 := \frac{I1_{zz}}{I1_{zz} + m_1 \cdot h_1^2}$$

$$C2 := \begin{bmatrix} 18 \\ 37 \\ 45 \\ 33 \\ 9.0 \\ 0.0 \\ 0.0 \end{bmatrix} \cdot cm \qquad W2 := \begin{bmatrix} 0.0 \\ 36 \\ 72 \\ 108 \\ 144 \\ 180 \\ 216 \end{bmatrix} \cdot cm$$

$$m_2 := \frac{(3543) \cdot lb}{g} = 110.1\ \frac{lb \cdot s^2}{ft}$$ 09 Pontiac G6

$$I2_{zz} := 2318.66 \cdot lb \cdot ft \cdot sec^2$$ From 4N6Xpert Autostats

$$h_2 := 1.1 \cdot m$$

$$\gamma_2 := \frac{I2_{zz}}{I2_{zz} + m_2 \cdot h_2^2} \qquad e := 0.0 \qquad i := 0..5$$

$$CDRlong2 := -7.46\ mph$$
$$CDRlat2 := 17.62 \cdot mph$$

$$CDRtotal2 := \sqrt{CDRlong2^2 + CDRlat2^2} = 19.1\ mph$$

$$\Pi_{PDOF2} := \operatorname{atan}\left(\frac{CDRlong2}{CDRlat2}\right) = -22.9\ deg$$

$$C2\_bar := \frac{\sum_{i=0}^{5} \left(W2_{i+1} - W2_i\right) \cdot \left(\frac{C2_i + C2_{i+1}}{2}\right)}{W2_6} = 8.7 \; in$$ weighted average deformation

$\Delta W_{scrape} := W2_6 - W1_6 = 44.9 \; in$ scraping distance on vehicle 2

$\mu_k := 0.5$ scraping friction coefficient

$\mu_r := 0.80$ surface friction coefficient asphalt

$n_2 := 75\%$ brake efficiency (broadside rear rotate)

$$F1 := \sum_{i=0}^{5} \left(A1 + B1 \cdot \left(\frac{C1_i + C1_{i+1}}{2}\right)\right) \cdot \left(W1_{(i+1)} - W1_{(i)}\right) \cdot \left(\frac{1}{\cos\left(\Pi_{PDOF1}\right)}\right) = \left(3.4 \cdot 10^4\right) \; lb$$

Matched Segment Piecewise Analysis:

$$E1 := \left(\sum_{i=0}^{5} \left(A1 \cdot C1_i + \frac{B1}{2} \cdot \left(\frac{C1_i + C1_{i+1}}{2}\right)^2 + \frac{A1^2}{2 \cdot B1}\right) \cdot \left(W1_{i+1} - W1_i\right) \cdot \left(1 + \tan\left(\Pi_{PDOF1}\right)^2\right)\right) = \left(1.4 \cdot 10^4\right) \; lb \cdot ft$$

$$E2 := \sum_{i=0}^{5} F_i \cdot \left(\frac{C2_i + C2_{i+1}}{2}\right) \cdot \left(1 + \tan\left(\Pi_{PDOF2}\right)^2\right) = \left(3.3 \cdot 10^4\right) \; lb \cdot ft$$

$$E1_{friction} := m_1 \cdot g \cdot \mu_k \cdot \left(\frac{m_2}{m_1 + m_2}\right) \cdot W1_6 = \left(3.2 \cdot 10^3\right) \; lb \cdot ft$$

$$E2_{friction} := m_2 \cdot g \cdot \mu_k \cdot \left(\frac{m_1}{m_1 + m_2}\right) \cdot \Delta W_{scrape} = \left(3.6 \cdot 10^3\right) \; lb \cdot ft$$

$$E1_{Gen} := E1 + E1_{friction} = \left(1.7 \cdot 10^4\right) \; lb \cdot ft$$

$$E2_{Gen} := E2 + E2_{friction} = \left(3.7 \cdot 10^4\right) \; lb \cdot ft$$

$$\Delta tpeak := \sqrt{\left(\frac{2 \cdot m_1 \cdot \gamma_1 \cdot m_2 \cdot \gamma_2}{m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2}\right) \cdot \frac{E1_{Gen} + E2_{Gen}}{F1^2}} \qquad \Delta tpeak = 0.06 \; s$$

$$\Delta v1 := (1 + e) \cdot \sqrt[2]{\left(\frac{2 \cdot m_2 \cdot \gamma_2 \cdot \left(E1_{Gen} + E2_{Gen}\right)}{m_1 \cdot \gamma_1 \cdot \left(m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2\right)}\right)} - \frac{m_2 \cdot \left(g \cdot \mu_r \cdot n_2\right) \cdot \Delta tpeak}{m_1} = 15.1 \; mph$$

$$\Delta v2 := (1 + e) \cdot \sqrt[2]{\left(\frac{2 \cdot m_1 \cdot \gamma_1 \cdot \left(E1_{Gen} + E2_{Gen}\right)}{m_2 \cdot \gamma_2 \cdot \left(m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2\right)}\right)} - \left(g \cdot \mu_r \cdot n_2\right) \cdot \Delta tpeak = 19.7 \; mph$$

CDR = 19.1 mph

Weighted Average Deformation Depth Analysis:

$$E2 := F1 \cdot C2\_bar \cdot \left(1 + \tan\left(\Pi_{PDOF2}\right)^2\right) = \left(3 \cdot 10^4\right)\ lb \cdot ft$$

$$E1_{Gen} := E1 + E1_{friction} = \left(1.7 \cdot 10^4\right)\ lb \cdot ft$$

$$E2_{Gen} := E2 + E2_{friction} = \left(3.3 \cdot 10^4\right)\ lb \cdot ft$$

$$\Delta tpeak := \sqrt{\left(\frac{2 \cdot m_1 \cdot \gamma_1 \cdot m_2 \cdot \gamma_2}{m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2}\right) \cdot \frac{E1_{Gen} + E2_{Gen}}{F1^2}} \qquad \Delta tpeak = 0.057\ s$$

$$\Delta V1 := (1+e) \cdot \sqrt[2]{\left(\frac{2 \cdot m_2 \cdot \gamma_2 \cdot \left(E1_{Gen} + E2_{Gen}\right)}{m_1 \cdot \gamma_1 \cdot \left(m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2\right)}\right)} - \frac{m_2 \cdot \left(g \cdot \mu_r \cdot n_2\right) \cdot \Delta tpeak}{m_1} = 14.6\ mph$$

$$\Delta V2 := (1+e) \cdot \sqrt[2]{\left(\frac{2 \cdot m_1 \cdot \gamma_1 \cdot \left(E1_{Gen} + E2_{Gen}\right)}{m_2 \cdot \gamma_2 \cdot \left(m_1 \cdot \gamma_1 + m_2 \cdot \gamma_2\right)}\right)} - \left(g \cdot \mu_r \cdot n_2\right) \cdot \Delta tpeak = 19.0\ mph$$

CDR = 19.1 mph

This example application to a real-world collision event demonstrates the G-DaTAΔ*V*™ System of Equations accurately determines the *total velocity change* even for a collision with significant inter-vehicular friction due to scraping, which would have been difficult to reconstruct reliably with previous vehicle deformation analysis methods.

G-DaTAΔ*V*™ System of Equations Limitations

Every model developed and intended to evaluate the behavior of a mechanical or physical condition is an approximation no matter how precise, detailed, or descriptive. Therefore, it is important to evaluate such models for accuracy through application comparisons with applicable testing. The RICSAC and NASS evaluations of the G-DaTAΔ*V*™ System of Equations provide the comparative assessment of the accuracy, precision, and efficacy of the approximations of *total velocity change* for non-central impacts — when analyzing vehicle deformation profiles utilizing the derived algorithms[1]. Regardless of the relative degree of accuracy, it is equally important to determine where variable sensitivities to the accuracy of the approximations may exist. As a result of analyzing the RICSAC and NASS data, general observations regarding variable sensitivity while applying the G-DaTAΔ*V*™ System of Equations are as follows[11]:

- The G-DaTAΔ*V*™ System of Equations is not sensitive to reasonable random and/or systematic differences between collision deformation measurements obtained by different, properly trained investigators. Differences in deformation depth measurements of ±10% generally resulted in no more than a ±2% difference in the *total velocity change* results for all RICSAC tests combined. The greatest deviation for a systematic increase or decrease in deformation depth measurements for both involved vehicles of ±10% was a difference in *total velocity change* of ±8.6% (greatest deviation in RICSAC 2).

- The G-DaTAΔ*V*™ System of Equations is not sensitive to the inertial properties approximated by using commercially available data in the absence of directly measured vehicle weights and weight distributions. Varying vehicle masses by ±10% resulted in approximately a ±3.1% difference in *total velocity change* results across the board for all RICSAC tests.

- The G-DaTAΔ*V*™ System of Equations is not sensitive to the choice of A and B stiffness coefficients obtained through the NEI database[7], as long as they are for the appropriate impacted surface (i.e., front, rear, or side), and the test is for sister vehicles that are within the manufacture year range for the same vehicle or its corporate clones. Varying A/B stiffness values by ±10% resulted in approximately a ±3.1% difference in *total velocity change* results across the board for all RICSAC tests.

- The G-DaTAΔ*V*™ System of Equations is not sensitive to the effects of inter-vehicular friction, since the majority of the work/energy contributions from this effect are quite small as compared to the work done by the impact impulse. Varying inter-vehicular friction values by ±20% from a default m_k = 0.5 g produced no more than a ±1.1% difference in *total velocity change* results (greatest deviation in RICSAC 2). However, ignoring inter-vehicular friction for collisions with scraping of 0.75 m (30 inches) or more resulted in an under-approximation of *total velocity change* by as much as -9.4% (greatest deviation in RICSAC 6).

- The G-DaTA*ΔV*™ System of Equations is not sensitive to the choice of drag factor for the roadway as long as the chosen drag factor is within reason for the particular roadway surface; i.e., asphalt, concrete, dry, wet, etc. Varying roadway friction or braking efficiency values by ±20% generally resulted in no more than a ±1.5% difference in total velocity change calculations (greatest deviation in RICSAC 1). Ignoring braking effects for broadside offset and oblique impacts resulted in errors in *total velocity change* up to approximately ±6.1% (greatest deviation in RICSAC 1).

The most critical elements of the G-DaTA*ΔV*™ System of Equations having the greatest potential for affecting the accuracy of the *total velocity change* approximations lay in the determination of the restitution coefficient, the PDOF acting upon each vehicle during the impact, and the resultant moment arm about the vehicle mass centers. The PDOF angle contribution affects the total deformation depth and, therefore, the total work due to impact forces. Additionally, the direction and location of the application of the PDOF determines the moment arm created by an applied force offset from the vehicle mass center and thus the rotational contributions to the *total velocity change* resulting from a non-central impact condition[11].

- Neglecting restitution may produce as much as a -19.8% under-approximation of the *total velocity change* for the vehicle of the least mass with respect to collisions involving impacts with wheels, tires, and axles where the coefficient of restitution ranges from e = 0.2 to 0.4 (greatest deviation in RICSAC 3).

- Ignoring the principal direction of force correction to the deformation depth produced as much as an -33.0% effect upon the determination of *total velocity change* (greatest deviation in RICSAC 2).

- Ignoring the dynamic mass ratio rotational effects can result in as much as a -24.4% effect (greatest deviation in RICSAC 8) upon the *total velocity change* determination, with the most significant influence associated with oblique impacts with a moment arm approaching 1 m.

- As demonstrated in the original CRASH analysis of the RICSAC data[1,11], errors as high as 79.2% (greatest deviation in RICSAC 7) resulted when the PDOF adjustment, dynamic mass ratio for rotation, restitution, inter-vehicular friction, and tire/ground force contributions were neglected.

If a collision event results in a non-central configuration, the following steps should significantly reduce systematic errors introduced into the G-DaTA*ΔV*™ System of Equations[11]:

- Produce scaled diagrams of the vehicles and damage profiles resulting from the impact, including contact and induced damages.

- Position colliding vehicles together at either initial contact or at maximum engagement for determining the location and direction of the PDOF application upon each vehicle, as demonstrated by **Figure 3**.

- Unless accurately and precisely determined, range the measured values for the PDOF angle and the moment arm for determining the effective rotational (dynamic) mass ratio, *g*, for both vehicles.

- Unless directly measured, range the effective roadway net drag factor when tire/ground impulse contributions should be considered.

Following these simple procedures when determining the *total velocity changes* and time to peak force application (used for determining peak accelerations), random and/or systematic errors should be significantly reduced, providing the forensic engineer with reasonable confidence in the accuracy and precision of the G-DaTA*ΔV*™ System of Equations.

Application of the G-DaTA*ΔV*™ System of Equations with respect to the RICSAC and NASS data also revealed the following observations regarding collision restitution considerations[11]:

- For high-speed collisions producing deformation depths averaging 0.3 m (12 inches) or more over the deformation width, ranging restitution between $0 \le e \le 0.10$ will provide accurate consideration of restitution effects.

- For impacts into the front or rear wheels/axles of at least one vehicle, even when deformation is significantly greater than 0.3 m, a restitution range between $0.2 \le e \le 0.40$ will provide accurate consideration of restitution effects.

- Low velocity impacts where the total velocity change is within the range $0 < Dv^{Total} \le 4.5$ m/sec (approximately 10 mph), the restitution will vary between $e = 0.6$ at very low velocities to $e = 0.3$ at the upper levels of the low velocity range. Selection of an appropriate restitution value is often an iterative process, but ranging the restitution is expected to provide greater assurance of an accurate consideration of restitution effects.

In all of the RICSAC and NASS collisions analyzed, an inter-vehicular friction of $m_{scrape}=0.5$ was used and did not vary between analyses. However, if evidence of snagging between the sliding surfaces is present, such as body panels pulled in the direction of sliding between vehicles, then consideration of higher inter-vehicular friction values may be appropriate. Again, ranging inter-vehicular friction for snagging conditions is likely to produce greater accuracy, but less precision in the analysis results. However, the contribution of inter-vehicular friction is the least significant of all other energy sinks or impulse and rotational contributions to *total velocity change*.

The damage analysis methods in existence previous to those developed by this engineer required knowledge of structural stiffness data for both vehicles involved in a given collision event, which limited their application with regard to many real-world collision events. However, the elimination of the need for the A and B stiffness data for one of the involved vehicles in a collision event allows for a much broader application to include collisions involving vehicle classifications with limited or no structural stiffness data. The major limitation of the G-DaTAΔ*V*™ System of Equations is that it remains reliant upon full-scale impact testing for determining the A and B structural stiffness values for one of the involved vehicles. The continued reliance upon structural stiffness values requires continued full-scaled impact testing, and may require testing of non-conventional vehicles or impact conditions when structural stiffness data for at least one of the vehicles is not available.

Additional limitations to the applications of the G-DaTAΔ*V*™ System of Equations result when vehicle deformation profiles cannot be reasonably measured directly or indirectly through photographic evidence — or when the analyst has limited training or understanding regarding proper deformation profile measurements. However, even though the G-DaTAΔ*V*™ System of Equations are not particularly sensitive to minor deformation profile measurement fluctuations, unrealistic approximations of deformation width and/or depth will have an effect upon the accuracy of the model. Damage profile width and depth determination is quite intuitive, and is also the subject of collision investigation training courses. The NASS data analysis demonstrates that random differences in measurement of deformation profiles between properly trained investigators, outside of those individuals that are intentionally biasing measurements, do not produce significant random errors that tangibly affect the analysis of *total velocity change* when using the G-DaTAΔ*V*™ System of Equations.

If critical variables are unknown or cannot be reasonably approximated, the use of the G-DaTAΔ*V*™ System of Equations may be limited or unreliable. Proper engineering judgement should be exercised when applying these algorithms or any other form of analysis to a real-world or staged collision event, if the determination of critical variables is complicated by other factors or if there is uncertainty in their reliability.

Conclusions

Correlation and descriptive statistics, as well as the raw analysis results, indicate that a reliable and significantly improved degree of precision and accuracy was achieved when the G-DaTAΔ*V*™ System of Equations were applied to both the RICSAC-staged collision tests presented in an earlier paper[1] and the NASS real-world collision data (when determining the total velocity changes for oblique and offset non-central impacts). Unlike the 12 RICSAC tests, the NASS real-world collisions were not carefully staged and instrumented, nor were the many variables input into the G-DaTAΔ*V*™ System of Equations documented to the precision of the RICSAC tests. Additionally, the RICSAC testing documentation was completed by the same team of researchers at one test facility, while each NASS real-world collision occurred at varying locations across the United States over a three-year span, and were each documented by different NASS-trained investigators. The NASS real-world collision data set represents a realistic comparison to field-collected data that a collision investigator could encounter when tasked with reconstructing an actual collision event. Stark and pronounced differences between the results of applying the G-DaTAΔV™ System of Equations to the controlled RICSAC staged testing (versus the random NASS documented real-world collisions) should have been present if the algorithms developed within this study had systematic errors, violations in the physics of oblique and offset non-central impacts, or significant sensitivity to analysis variable inputs. Instead, the results of the RICSAC and NASS evaluations using the G-DaTAΔV™ System of Equations demonstrate an expectation of a reasonable degree of data correlation, accuracy, precision, and efficacy when applied to real-world collision events.

G-DaTAΔV™ System of Equations provides an accurate and reliable tool for the forensic engineer to determine the *total velocity change* levels produced by real-world collision events. The presented methods have been applied by this author to the following impacts where vehicle and surface specific structural stiffness characteristics were either scarce or non-existent:

- Broadside or oblique side impacts.
- Rear end impacts.
- Impacts involving light trucks, vans, and sport utility vehicles where vehicle and surface-specific structural stiffness values are scarce.
- Impacts involving heavy vehicles, buses, RVs, motorcycles, and other similar vehicles with few vehicle and surface-specific data.
- Impacts with non-vehicular objects, or unique vehicles such as trailers or heavy equipment that deform when struck, but have no known structural stiffness data.

Future research should include further validation when applying the G-DaTAΔV™ System of Equations to commercial vehicles, motorcycles, and trailers.

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