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G-Force Analysis for Low-Speed Rear-End Collisions

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Abstract

Low-speed rear-end collisions cause a Delta-Velocity (Delta-V) of less than 5 MPH or so, but often result in litigation because of claims for "whiplash" type of soft tissue injuries. Forensic consulting engineers are then called to estimate the occupant g-force but find there is only minor or, possibly, no visible damage to the vehicles. Nevertheless, the worst-case Delta-V and g-force can generally be estimated by comparing the impact damage, or lack of impact damage, with published low-speed crash test data or, in many cases, by engineering analysis. This paper will discuss a variety of approaches to analyze the g-force and will identify some useful data sources to support the comparative analyses.

The Role of the Forensic Engineer

For a low-speed rear-end collision, the ultimate goal of the Forensic Engineer is, usually, to determine the maximum g-force on a particular occupant in the vehicle that was struck from behind. The engineer will evaluate the evidence for the collision, try to envision the collision characteristics that best fit all the evidence and, then, perform an analysis to verify the vision scenario and to determine significant collision parameters. Evaluation of the evidence will, generally, be based upon (1) evaluation of file data for the case, e.g., police reports, depositions, repair estimates, medical data, witness statements, etc., (2) personal inspection of the vehicles and the accident site, and (3) technical data from standard engineering reference sources. The analytical procedures will be based on accepted standards of engineering analysis and may include, as required, principles of conservation of energy and momentum, equilibrium of forces and moments, strength and bending of materials, kinematic analysis, and force-mass-acceleration-time relationships.

Engineering Analysis –

Impact Time Duration: In some analyses it may be necessary to assume a time duration for the impact and, generally, the time duration for a low-speed rear-end collision will be somewhere in the range of 100 to 150 milliseconds. When it is necessary to estimate an impact time duration, the engineer should use some logic and judgment to select the appropriate time duration but, in gen-

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eral, the lower the approach velocity and the "softer" the bumper mounting components, then the longer will be the impact time duration.

Three Approaches for the Engineering Analysis: If the impact force or the Delta-V can be estimated for either vehicle in a low-speed rear-end collision, at least for the worst-case scenario, then the g-force can also be estimated for the occupants. The impact force or Delta-V to the vehicle can generally be estimated by one of three approaches:

1. Analyze Mechanical Evidence: Careful inspection of potential damage areas on the vehicles may reveal some type of physical evidence, e.g., bent beams, nut or bolt slippage patterns, energy absorber stroke marks, etc. Then the impact force or Delta-V can often be estimated by analyzing the mechanical characteristics that will reproduce the same physical evidence.

2. Compare Damage to Known Crash Test Data: The impact damage, or lack of impact damage, can often be compared with published low-speed crash test data to estimate the probable Delta-V. A few publications that provide useful low-speed crash test data will be identified below to help with these comparisons.

3. Analyze Indirect Evidence: If no physical evidence of the accident can be found in the bumper systems and the published crash test data indicate that neither vehicle was damaged in low-speed crash tests, then the forensic engineer must find some other means to estimate the g-force in the accident. Surprisingly, a credible engineering analysis can, generally, be performed to determine the probable characteristics of the accident based on indirect evidence from the inspections and file data.

Scope of This Paper

Each of the three analytical approaches will be discussed and several simple "Case In Point" examples will be given to demonstrate the analysis procedures. First, a case with impact evidence (measured striation marks on viscous-damper type of energy-absorber bumper mounts) will be used to evaluate the worst-case Delta-V to a vehicle and g-force on the driver. Second, a case with no significant bumper system damage to either vehicle will be compared with low-speed crash test data to estimate the worst-case Delta-V to a vehicle and g-force to the occupants. And, third, a few examples will be shown for the use of indirect evidence (idling acceleration, maximum acceleration, and average reaction time) to estimate the Delta-V and g-force. However, before starting with the analyses, some discussion of Low-Speed Rear-End crash characteristics plus some discussion of useful formulas and reference data will be presented.

Effect of Energy Storage and Restitution

Elastic Restitution Will Increase the Delta-V of Both Vehicles: Restitution is an effect caused by elastic bending and compression of bumper system components whereby, after maximum impact penetration has occurred (the point at which both vehicles will have a common velocity), the components will begin to rebound and give their absorbed energy back to the vehicles - in the same manner that a compressed spring can give back its stored energy. This effect can be seen in Figures 1 and 2, taken from a recent series of 33 low-speed rear-end collisions staged by the Southwestern Association of Technical Accident Investigators (SATAI) in Phoenix, AZ.[1] These tests were supervised and analyzed by MacInnis Engineering Associates, Ltd., and the figures are used herein by permission of the first author. The figures show the velocity histories for impact of a 1984 Toyota Van (bullet vehicle) into the rear of a 1986 Buick Century (target vehicle). The effect of restitution can be seen after the point of maximum penetration has been reached. The restitution causes the bullet (striking) vehicle to continue slowing down while causing the target (struck) vehicle to continue speeding up. The elastic components are giving their stored energy back to the vehicles by trying to push them apart during the restitution phase and the elastic force on each vehicle will be equal but in opposite directions. Note that the acceleration rates (slope of the velocity curves) are higher during the initial impact phase than during the restitution phase. In other words, the effect of restitution is to increase the Delta-V of each vehicle but, generally, at an acceleration rate below that of the initial impact phase.

Elastic Restitution Depends on the Specific Bumper Systems and Impact Speed: The coefficient of restitution (ε) is defined as the ratio of the relative velocities before and after a collision, that is:

$$\varepsilon = -\left(V_B' - V_T' \right) / \left(V_B - V_T \right) \qquad \{1\}$$

where V_B and V_T are the velocities of the Bullet and Target vehicles before impact, and V_B' and V_T' are their velocities after impact. A plastic collision, for example, has a coefficient of 0.0 while a perfectly elastic collision will have a coefficient of 1.0. Most vehicle impact cases will, obviously, have a coefficient somewhere in between. The coefficient of restitution for the impact of the Toyota Van into the rear of the Buick Century at a closing speed of 2.7 and 6.2 MPH can be determined from Figures 1 and 2, respectively, that is:

$$\varepsilon_{2.7} = -\left(0.7 - 1.9 \right) / \left(2.7 - 0.0 \right) = 0.4 \qquad \{2\}$$

$$\varepsilon_{6.2} = -\left(1.9 - 3.4 \right) / \left(6.2 - 0.0 \right) = 0.2 \qquad \{3\}$$

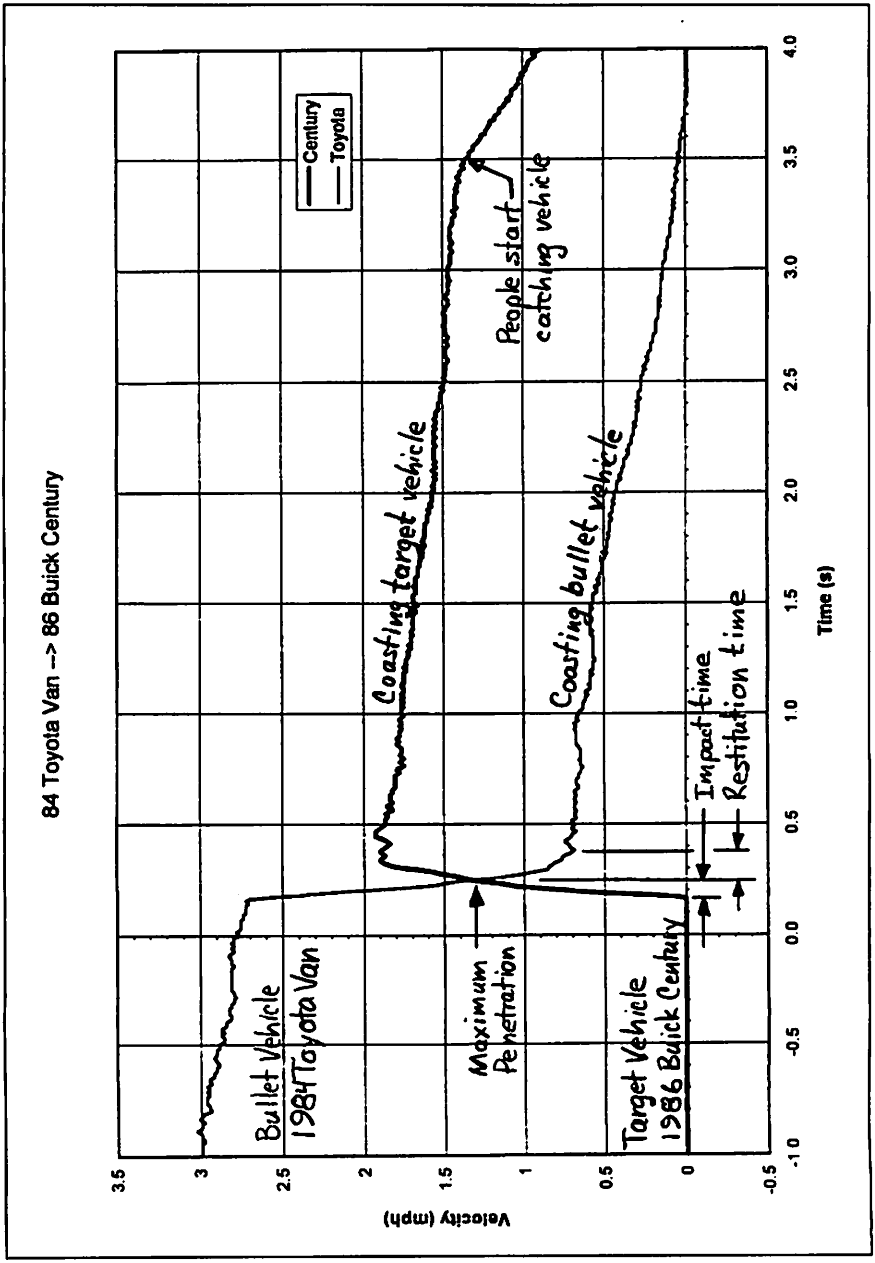


Figure 1

Velocity Histories for Impact of a 1984 Toyota Van Into a 1986 Buick Century at a Closing Velocity of 2.7 MPH.

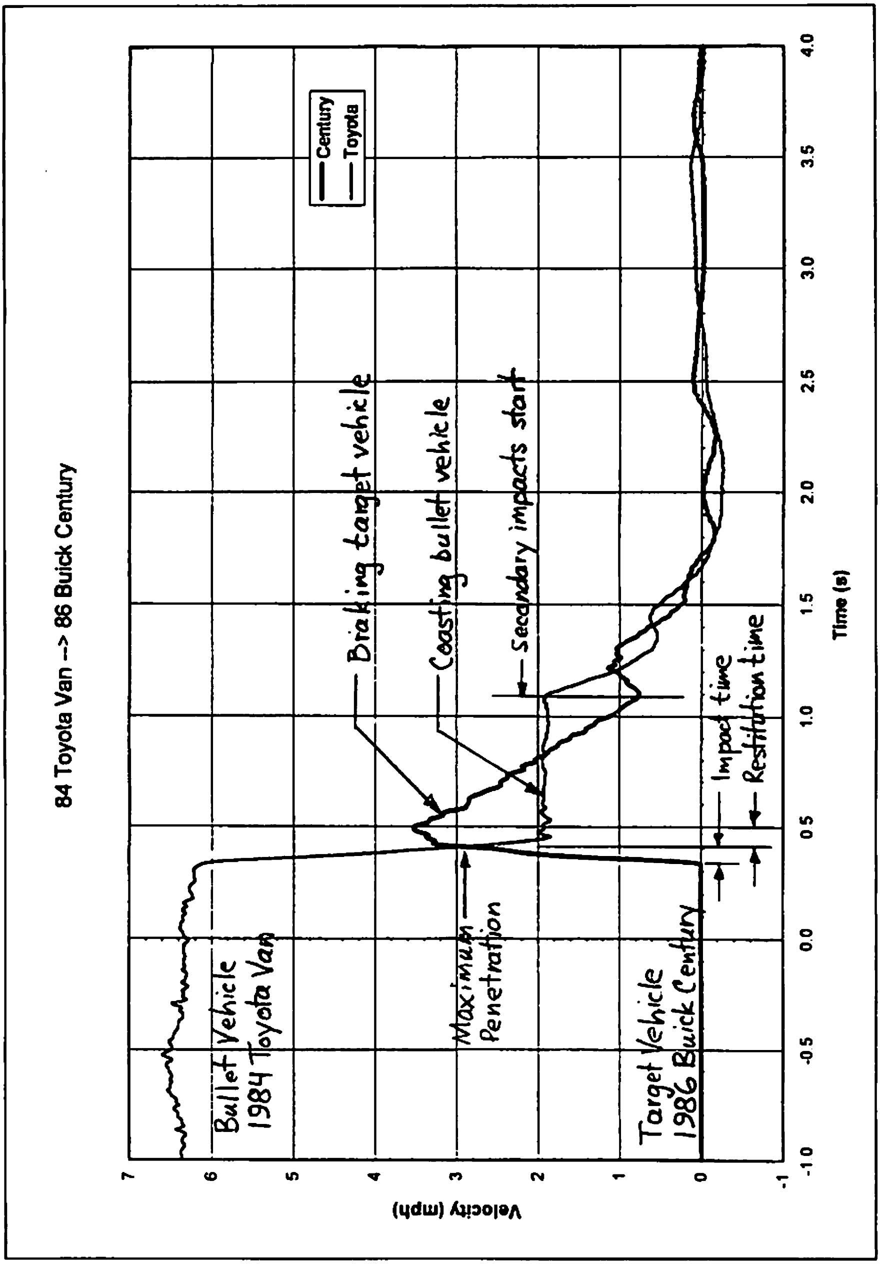


Figure 2

Velocity Histories for Impact of a 1984 Toyota Van Into a 1986 Buick Century at a Closing Velocity of 6.2 MPH.

In Figure 1, the maximum penetration occurred at a common velocity of 1.3 MPH. However, because of restitution, the velocity of the Toyota decreased further, from 1.3 to 0.7 MPH (- 47%) while that of the Buick increased from 1.3 to 1.9 MPH (46%). In Figure 2, the maximum penetration occurred at a common velocity of 2.8 MPH. Because of restitution, the velocity of the Toyota decreased further, from 2.8 to 1.9 MPH (- 32%) while that of the Buick increased from 2.8 to 3.4 MPH (21%). In other words, the coefficient of restitution depends on the specific bumper systems and on the impact velocity of the two vehicles. If this effect becomes important to an analysis, MacInnis Engineering Associates can, possibly, supply specific information from their data base of staged rear-end, front-end and lateral collisions. To date they have performed 2500 such crash tests and are currently working on a plan to make that data available to the public for a fee.[1]

Elastic Restitution May Not Be Critical to an Analysis: From an engineering standpoint, it is important to understand the possible effect of elastic restitution but the effect of elastic restitution may not be critical for the determination of probable injury. It is easy to understand that, for a very light impact, nearly all the kinetic energy will be absorbed by elastic bending. As the impact intensity increases, the vehicle components will continue to store energy but the relative energy storage effect becomes smaller and smaller. As the impact velocity increases, more and more of the kinetic energy is dissipated by permanent yielding of the components, sheet metal and frame members, i.e., by plastic deformation. For high-speed collisions, the relative effect of restitution is very small and it is generally neglected. For very low-speed collisions (with a Delta-V on the order of 1 - 2 MPH, as for Figure 1) the effect of restitution can change the Delta-V values by 50% or so, i.e., by 0.5 to 1 MPH. However, the g-force is already so low in such cases that this effect is not critical to the determination of probable injuries. For low-speed collisions with a Delta-V of 5 MPH, or so, the effect of restitution can change the Delta-V by 10% - 20% or so, which is also 0.5 to 1 MPH. This effect, again, may not be critical to the determination of probable injury because (1) the restitution effect is beyond the order of accuracy of many other parameters in the analysis, and (2) the time duration of the restitution is generally longer than that of the initial impact so the g-force (acceleration rate = slope of the velocity vs. time curve) is generally lower during the restitution than it is during the initial impact phase of the collision. For simplicity, the effect of elastic restitution will not be included in the velocity relationships derived below.

Velocity Relationships for Low-Speed Rear-End Collisions

General Case for a Collision Between Two Moving Vehicles: First, it should be made clear that the Delta-V of each vehicle in a collision is not dependent on the absolute velocity of either vehicle. Rather, the Delta-V depends only on their relative velocities. For example, when two vehicles are traveling at highway speeds and one hits the other from behind, the collision may still be a low-speed rear-end collision if the Delta-V of the vehicles is less than 5 MPH or so. This is just a more general case of the common analysis where one vehicle is stationary.

Velocity Relationships: If two moving vehicles of weight W_B and W_T have initial velocities of V_B and V_T, respectively, where $V_B \geq V_T$, then the impact diagram for the collision will be as shown in Figure 3. Because the effect of restitution has been omitted, the two vehicles will have a common velocity after the collision. At that point, the time duration of the collision is essentially completed. The relationships between the velocity and Delta-V of the two vehicles can be defined as:

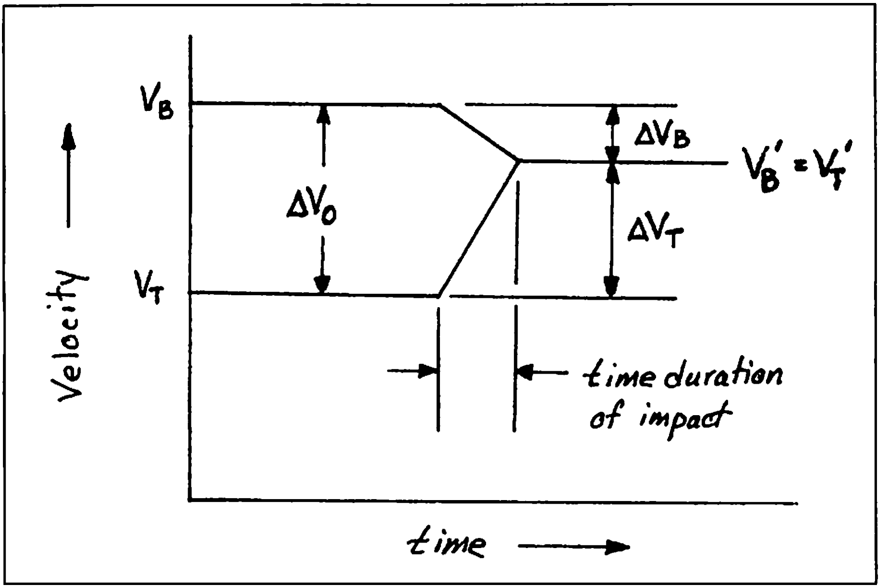


Figure 3
An impact diagram to identify the relative velocity relationships between two vehicles, B and T, during a rear-end collision (V = Velocity and t = time).

$$V_B \equiv V_T + \Delta V_0 \quad \{4\}$$

where ΔV_0 is the initial velocity difference or closing speed between the vehicles.

Note: By definition, velocity is a vector quantity which designates speed and direction. However, it is normal practice in the industry to use the words velocity and speed interchangeably (as a scalar value) because it is easier, i.e., more intuitively correct, to understand the dynamics of the collision, especially in view of the relative velocity relationships of Figure 3. This intuitive practice will be followed in the equations derived below. For those more mathematically inclined, just replace the quantity (ΔV_B) by the quantity (- ΔV_B) in the equations below and the mathematics are then rigorously correct. The only difference between the intuitive approach and the mathematical approach is that the change in speed of vehicle B is, mathematically, negative because the vehicle decelerates during the collision.

The common velocity of the vehicles after impact will be:

$$V_B' = V_T' = (V_B - \Delta V_B) = (V_T + \Delta V_T) \qquad \{5\}$$

where

$$\Delta V_0 = \Delta V_T + \Delta V_B \qquad \{6\}$$

and where ΔV_B and ΔV_T are the scalar values of the change in velocity (Delta-V) of vehicles B and T, respectively. By Conservation of Linear Momentum:

$$W_B V_B + W_T V_T = W_B (V_B - \Delta V_B) + W_T (V_T + \Delta V_T) \qquad \{7\}$$

$$\text{or} \quad W_B \Delta V_B = W_T \Delta V_T \qquad \{8\}$$

Then, by substituting from equation {6}:

$$\Delta V_B = \Delta V_0 [W_T / (W_B + W_T)] \qquad \{9\}$$

and

$$\Delta V_T = \Delta V_0 [W_B / (W_B + W_T)] \qquad \{10\}$$

The total Kinetic Energy "lost" during the impact, i.e., absorbed by the bumper energy absorbing system and by plastic deformation, can be defined as ΔKE where:

$$\begin{aligned} \Delta KE &\equiv \text{Initial KE - Final KE} \qquad \{11\} \\ &= [(W_B V_B^2 + W_T V_T^2) /2g] - [W_B(V_B-\Delta V_B)^2 + W_T (V_T+\Delta V_T)^2]/2g \\ &= W_B \Delta V_B [V_B+ (V_B-\Delta V_B)] / 2g - W_T \Delta V_T [V_T + (V_T + \Delta V_T)]/2g \end{aligned}$$

Then by substituting from equations {5} and {8}, and from equations {9}, {10} and {4}:

$$\begin{aligned} \Delta KE &= (W_B V_B)\,\Delta V_B / 2g - (W_T V_T)\,\Delta V_T / 2g \qquad \{12\} \\ &= (V_B - V_T)\,\Delta V_0\,[\,W_B W_T / (W_B + W_T)] / 2g \\ &= W_B\,\Delta V_0^2\,[W_T / (W_B + W_T)] / 2g \end{aligned}$$

In other words, the total Kinetic Energy "lost" during the impact is directly related to the relative weight of the vehicles.

1. Special Case: Single Vehicle Impact Into a Fixed Barrier: If a single moving vehicle collides with a fixed barrier, then $W_T \equiv \infty$, $V_T \equiv 0$, and from equation {4}, $V_B = \Delta V_0$, so equation {12} reduces to the familiar expression:

$$\Delta KE = W_B V_B^2 \{ 1 / [(W_B / \infty) + 1] \} / 2g = W_B V_B^2 / 2\,g \qquad \{13\}$$

In other words, the Kinetic Energy "lost" in a fixed barrier collision is simply equal to the total initial Kinetic Energy of the vehicle. For a low-speed collision, the relatively low Kinetic Energy of the vehicle will be absorbed by compression of the bumper energy absorbing system rather than by crushing of body panels and frame members.

2. Special Case: Moving Mass Impact Into a Stationary Vehicle: If a moving mass B (such as a pendulum, rolling mass or another vehicle) collides with a stationary vehicle T, then $V_T \equiv 0$ and, from equation {4}, $V_B = \Delta V_0$. From equation {9}, the Delta-V of the moving mass will be:

$$\Delta V_B = \Delta V_0\,[\,W_T / (W_B + W_T)] = V_B\,[\,W_T / (W_B + W_T)] \qquad \{14\}$$

and, from equation {10}, the Delta-V of the vehicle will be:

$$\Delta V_T = \Delta V_0\,[\,W_B / (W_B + W_T)] = V_B\,[\,W_B / (W_B + W_T)] \qquad \{15\}$$

If the weight of the moving mass and the vehicle are the same, i.e., $W_B = W_T$, then:

$$\Delta V_B = \Delta V_T = (1/2)\,V_B \qquad \{16\}$$

and the common velocity of the moving mass and vehicle after the impact will be:

$$V_B - \Delta V_B = V_T + \Delta V_T = V_T + (1/2)\,\Delta V_0 = (1/2)\,\Delta V_0 = (1/2)\,V_B \qquad \{17\}$$

and the Kinetic Energy lost during the impact will be:

$$\Delta KE = (W_B V_B^2 / 2g)\,[W_T / (W_B + W_T)] = (W_B V_B^2 / 2g)\,[1/2] \quad \{18\}$$

In other words, if $W_B = W_T$ and $V_T = 0$ at impact, then 1/2 the initial Kinetic Energy will be lost. The Delta-V of the vehicle and that of the moving mass will be in opposite directions, the moving mass will decelerate while the vehicle will accelerate, but the scalar value of each will be equal to 1/2 the initial velocity of the moving mass.

Since the ΔKE from equation {18} is just 1/2 that of equation {13}, then it can be seen that (for a given weight, W_B, and velocity, V_B) the energy dissipated during an impact with a moving mass is only 50% of that dissipated during an impact with a fixed barrier. Or, in terms of velocity, the energy dissipation is the same for an impact against a moving barrier as it is for an impact against a fixed barrier at 71% of the velocity.

Some Useful Reference Data

Before proceeding with a discussion of the three analytical approaches some useful reference data will also be identified.

The Peak g-Force on Occupants in a Vehicle is About 0.9 g's per MPH of Delta-V: Allen, et al., of Wier-Jones Engineering Consultants, Ltd., indicates that measurements by Severy demonstrated that the peak acceleration of the head of human test subjects was about 0.5 g's/MPH of Delta-V.[2] Those tests were performed at impact velocities of 10 and 20 MPH in 1940's vintage cars without shock absorbing bumpers. Allen, et al., also indicated that unpublished case studies indicate the head acceleration was about 0.8 g's/MPH of Delta-V for collisions at less than 6.3 MPH (barrier equivalent impact) in modern vehicles with shock absorbing bumpers. Although these papers don't address the seat structure directly, the stiffness of the seat structure and design of the head restraint will certainly have an effect on the head acceleration levels. More recently, McConnell, et al., of Biodynamics Research Corporation performed low-speed rear-end crash tests with instrumented, healthy, human test subjects in both the target and bullet vehicles.[3] Data from their published test demonstrated that the maximum acceleration, or "peak" g-force, at the top of the cervical spine of a driver in the target vehicle is about 0.9 g's/MPH of Delta-V of the vehicle. This higher value will be used below to estimate the g-force on the occupants of any vehicle in which a known Delta-V has been determined.

The Threshold of Injury for Human Test Subjects is About 4 to 5 MPH Delta-V: The tests by McConnell, et al., also demonstrated that a Delta-V of about 4 to 5 MPH is probably at, or near, the typical human threshold for very mild, single event musculoskeletal cervical strain injury.[3] In a similar test program West, et al., of Baker Engineering, Ltd., found that rear-end impacts up to a Delta-V of 8 MPH can be tolerated without injury if proper head support is provided.[4] In a more recent study, Szabo, et al., of Biodynamics Engineering, Inc., has demonstrated that impacts with a Delta-V of 5 MPH resulted in no injury to instrumented human test subjects even though the volunteers, both male and female, ages 27 to 58 years, had various degrees of documented cervical and lumbar spinal degeneration.[5] These tests resulted in measured "peak" g-forces of 3.9 to 5.2 g's at the lumbar spine, 4.5 to 7.4 g's at the cervical spine, and 10.1 to 13.7 g's at the head position. The study concluded that "The impacts resulted in no injury to any of the human volunteers and no objective changes in the condition of their cervical or lumbar spines."

Acceptable g-Force on Human Subjects in Everyday Activities: The g-force for any accident can be compared to the typical g-force that humans experience in everyday activities. For comparison, the peak g-force or acceleration vector in the midsaggital plane of the head is shown below for various everyday activities as measured by Allen, et al.,[2] and by Rosenbluth of Automotive Systems Analysis, Inc. and Hicks of Lowell Hicks, Inc.[6] Since most of these activities are accepted by individuals without complaint, then the g-force associated with such activities will also, normally, be acceptable even if received in other activities:

Activity	**g's**
• Swinging head to left, i.e., looking from right to left[2]	0.6
• Traversing a parking lot speed-bump in a small pickup at 5 MPH[6]	0.6
• Startled reaction to an unexpected starter pistol[2]	0.9
• Traversing a 6.25" curb drop in a small pickup at 6 MPH[6]	1.0
• A quick look over your shoulder, as for a traffic check[2]	1.1
• Standing upright suddenly from a kitchen chair[2]	1.9
• Surprise head-bob rearward, in seated position, as if falling asleep[2]	2.2
• Sitting down onto a kitchen chair[2]	2.5
• A good uninhibited sneeze[2]	2.9
• A typical non-exaggerated cough[2]	3.5
• Unexpected crowd jostle (bump to left posterior shoulder) from behind[2]	3.6
• Skipping rope (28 year old woman)[6]	3.9
• A surprise hale and hardy slap on the posterior shoulder, with the subject prepared and anticipating the "greeting"[2]	4.1

• Vigorous surprise kick to back of wheeled office chair with subject seated[2]	4.3
• Skipping rope (7 year old girl)[6]	5.7
• Hopping off an 8" step and landing on both feet[2]	8.1
• Plopping passively backwards into a low-back office-type chair[2]	10.4
• Routine head blows to a boxer[2]	70

Approach I: Analysis of Impact Evidence

Striation Marks From Compression of Energy Absorbing Bumper Mounts: To demonstrate an analysis of impact evidence, the Delta-V for a vehicle equipped with Energy Absorber Devices (EADs) will be presented. The analysis will be based on the measured length of the striation marks left on the EADs during their compression stroke. There are several types of energy absorbing bumper mount systems used in the industry and some of these are discussed briefly in the old SAE Recommended Practice J1571 for *Inspection of Energy Absorbing Bumper Mounts.*[7] Some EAD designs can be analyzed in a straight forward manner so they will only be discussed briefly here. Such devices produce a force that can be related directly to the deflection of the EAD. These characteristics are found in most spring-loaded, positive-deformation, elastomeric-shear, and clamped friction-force types of devices. In addition, however, some vehicles have EADs wherein the force cannot be correlated directly to the deflection. Some General Motors bumpers, for example, may be mounted with a hydraulic type of EAD wherein the EADs are, essentially, viscous dampers. In these EADs, the force is not related to the deflection but, rather, to the rate, or velocity, of the EAD compression. An example analysis will be given for this type of EAD.

Analysis of EADs Wherein the Force is Related Directly to the Displacement: The force vs. displacement characteristic of this device is repeatable, within manufacturing and material tolerances, and can be measured in a lab. A spring-loaded EAD, for example, has a linear force vs. displacement characteristic and will generally return to its original shape after an impact, provided that the yield strength is not exceeded in any of the EAD materials, i.e., the device is not compressed beyond its design limit. Similarly, a positive-deformation, friction or elastomeric shear type of EAD will have a fairly repeatable force vs. displacement characteristic but that characteristic may not be linear. Some of these EADs are one-time use devices that do not return to their original shape or position after the impact.

Measurement of a spring-loaded or elastomeric type of EAD is generally non-destructive (a friction-force device may or may not be non-destructive), so the actual device can be used for lab measurements. On the other hand, measurements of a deformation type of EAD is always destructive so proper authorization should be obtained prior to conducting such tests with any litigation evidence. Since the deformation type of EAD will already have been permanently deformed during the accident, then it can only be used to measure the force at a displacement level beyond that received in the accident. An exemplar EAD can be used to measure the force required to duplicate the displacement of most devices. Once the force vs. displacement characteristics of the EADs are known, then the work performed on the EADs during the accident (energy absorption capability of the EADs) can be found and equated to the Kinetic Energy lost. From that, the relative impact velocity and Delta-V of the vehicles can be estimated.

Some Vehicles Are Equipped With Hydraulic EADs: The Energy Absorbing Devices (EADs) on some vehicles – if so equipped – have a piston that is compressed into a cylinder during the impact. The impact displacement of these devices can usually be determined readily by tell-tale striation marks, or scraped paint, made during the compression stroke. The striation marks may not exist around the full circumference of the piston so care should be exercised to ensure a thorough inspection. The striation marks on the left and right EADs will generally be of different lengths, depending upon where the center of impact was on the bumper. It is possible that only one of the EADs will have striation marks if the center of impact was outboard of one of the EAD bumper mounts. In that case, the far device will generally have no compression because the bending moments on the bumper system will try to "pull" that piston out of the cylinder. The age of the striation marks and the effect of overlaid marks should be considered carefully because the striation marks may or may not be related to the accident in question. A recent set of marks, or a set of marks on top of another set, may be sufficient to ensure a high probability that the specific marks are related to an accident in question but, not necessarily. As a worst case, the longest striation marks can be used and it can then be stated that the vehicle has never received a worse impact.

Delta-V Estimate for Bumpers with Viscous-Damper Type of EADs: The force required to compress a viscous-damper type of EAD depends upon the rate of compression rather than on the displacement itself. Therefore, the force is nearly independent of the displacement. However, the design force for the EAD can be used, along with the measured compression stroke, to estimate the worst-case Delta-V of the vehicle.

1. **General Motors EADs Are Designed For A 5 MPH Impact:** The Energy Absorbing Devices (EADs) manufactured by the Delco Chassis Division of General Motors are designed so that a 2" stroke of two

EADs will absorb the energy of the vehicle during a 5 MPH barrier impact.[8] A cutaway view of this type of EAD is shown in Figure 4. The EADs are tailored for the weight range of the vehicle in which they are installed. Each EAD has a 200 PSI preload of 90% Nitrogen and 10% Helium (for leak detection) and the working fluid is, basically, hydraulic fluid. The 200 PSI preload pressure acts against a small "free floating piston" within the EAD piston to ensure that, once stroked, the EAD piston will return to its extended position. Because of the preload, the impact force must exceed about 400# before the piston will begin to move. Once the piston has begun to move, it will continue to move with any piston load greater than 400# or, more appropriately, greater than the force exerted by the gas pressure as the gas is compressed in the piston. The maximum design stroke (displacement) of each EAD is 2.0" after which it will "bottom out." The piston length does not affect the operation of the EAD but, rather, is tailored to provide the desired "stand-off" distance of the bumper from the frame of the vehicle.

2. The EAD Force is Nearly Constant Throughout The Stroke: Each Energy Absorbing Device (EAD) actually operates like a velocity-sensitive viscous damper. The EAD has an orifice with a tapered metering pin so that the design "Force v. Displacement" curve is nearly flat throughout the impact except for a possible initial transient at the beginning of the stroke. As the piston is forced into a cylinder filled with hydraulic fluid, the displaced fluid flows back into the piston through an orifice in the end of the piston. The piston, itself, contains a smaller free-floating piston with the hydraulic fluid on one side and the preload gas chamber on the other. As the piston is stroked, the orifice size decreases (via the tapered metering pin) so that the flow resistance will increase as the velocity of the vehicle slows down. This provides a nearly constant resisting force throughout the full 2" stroke of the EAD piston. If the initial velocity of the vehicle is faster or slower than the 5 MPH design speed then, likewise, the resisting force of the EAD will be greater or less, respectively, than the design force, and the stroke of the piston will bottom out or be less, respectively, than the 2" design stroke for the piston.

3. EADs Will Produce a Design Deceleration of About 5 g's: Energy Absorbing Devices (EADs) are designed so that two units will absorb the total kinetic energy when the vehicle impacts a barrier at 5 MPH, i.e., the Delta-V is 5 MPH. The total kinetic energy of the vehicle is dissipated as work (force x displacement) on the EAD pistons as they are compressed into their cylinders. Therefore, equating the kinetic energy to the work dissipation, then the design force (F_D) is found to be proportional to the design weight (W_D) and to the square of the design

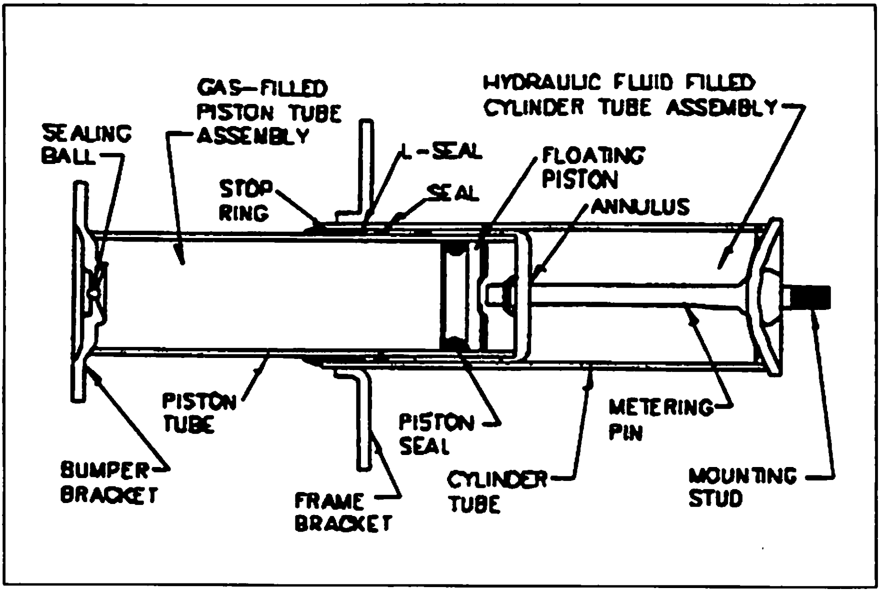


Figure 4
Cutaway view of a General Motors type of hydraulic Energy Absorbing Device (EAD). Cutaway drawing is courtesy of MacInnis Engineering Associates.[1]

Delta-V (ΔV_D = 5 MPH = 88 inches/second), and inversely proportional to the design stroke (l_D = 2.0") of the piston, thus:

$$F_D = W_D \, \Delta V_D^2 \, / \, 2g \, l_D \qquad \{19\}$$

where g is the acceleration due to gravity (g = 32.2 feet/second^2 = 386 inches/second^2). For example, if a vehicle with a 3000# design weight and 2" design stroke is stopped with a Delta-V of 5 MPH, then the average force in the EADs is about 15000#, or 7500# in each EAD. The design deceleration (a_D) for a GM vehicle equipped with hydraulic EADs can, also, be determined from the relationship, F = ma = (W /g) a, by substituting design values into the relationship, that is:

$$a_D = (F_D \, / \, W_D)g = (\Delta V_D^2/2g \, l_D)g = [88^2/(2 \times 386 \times 2)]g = 5g \quad \{20\}$$

In other words, the design deceleration for GM vehicles equipped with EADs is 5 g's. As a rough rule of thumb, the energy absorbed by a pair of these EADs can be determined from:

$$\Delta KE = (\Delta V_D^2 / 2g) W_D = (10.0") W_D \qquad \{21\}$$

In other words, the energy (in inch-pounds) absorbed by a pair of EADs is about 10 times the weight of the vehicle.

4. The Worst-Case Delta-V Of a Vehicle Can Be Estimated from the EAD Stroke: If the EADs on a vehicle have not bottomed out during an accident, then the worst-case Delta-V of the vehicle can be estimated by comparing the ratio of various design parameters to those same parameters as measured, or known, for the accident vehicle. For example, equation {19} can be written in terms of relative ratios as:

$$\Delta V^2 = \Delta V_D^2 (F/F_D) (W_D/W) (l/l_D) \leq \Delta V_D^2 (W_D/W) (l/l_D) \qquad \{22\}$$

where the subscript D refers to a design parameter and the non-subscripted parameters refer to the accident vehicle. The design parameters, $\Delta V_D = 5$ MPH and $l_D = 2"$, can be used, and the design weight (W_D) can be estimated by adding 50% of the specified load capacity to the curb weight of the vehicle. The EAD compression stroke (l) can be measured, and the actual weight of the vehicle (W) can be estimated by adding the estimated weight of the occupants and cargo to the curb weight of the vehicle. If the piston has not bottomed out, then the piston resisting force (F) will always be less than or equal to the design force (F_D), so the worst case velocity can be estimated by substituting ($F / F_D \leq 1$) into the equation, as shown. If two EADs have different stroke lengths, then the average of the two stroke lengths should be used. If only one EAD has resisted the impact, then the resisting design force (F_D) is only that from one EAD (7500# in the example above) so a force ratio of ($F / F_D \leq 1/2$) should be used. Thus, the worst-case Delta-V of the vehicle can be estimated from the length of the piston strokes (length of the striation marks) on the vehicle's EADs.

1. Case in Point: Worst-Case g-Force from Measured EAD Displacement:

Camaro Impacts Cavalier: A 2500#, 1988 Chevrolet Cavalier stopped at a light and was struck in the rear by a 3800#, 1979 Chevrolet Camaro. Four months later an attorney requested an investigation to estimate the g-force on the driver of the Cavalier.

The Evidence: The Camaro had been inspected and photographed by a professional appraiser. The front cowling had widespread damage. Personal inspection of the Cavalier revealed that the rear bumper was mounted with EADs and the right side of the bumper had several light scuff marks that could, possibly, be attributed to the accident. Photographs 1 to 4 show the left and right EADs on the Cavalier. The right EAD had 0.50" long striation marks that were of recent origin while the left EAD had no indication of any prior stroke.

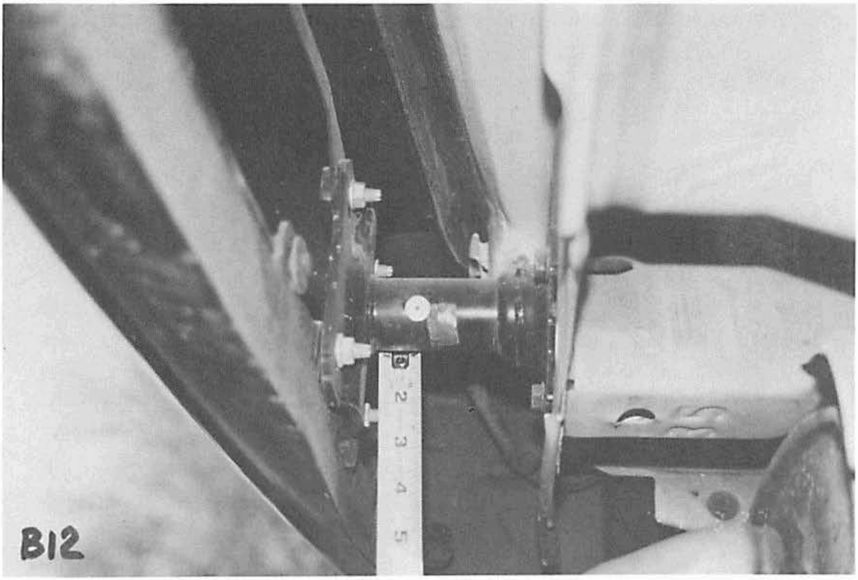
The Analysis: A Mitchell Collision Estimating Guide Domestic, General Motors, revealed that the front cowling of the Camaro covers the whole front end of the vehicle. Discussions with prior owners revealed that the front cowling had numerous prior impacts at the center and on both corners of the front bumper area. An inspection would have been of little value. Several specific scuff marks on the Cavalier were found to be at the expected height (to match the Camaro's bumper cover). The scuff marks were centered near the right bumper mount.

The Key: The matching scuff marks over the right bumper mount plus the EAD stroke on the right side indicated that the center of impact was over the right EAD. Therefore, only one EAD had resisted the impact so a force ratio of 1/2 was used in equation {22} and the worst-case Delta-V to the Cavalier turned out to be less than 1.9 MPH.

The g-Force: The maximum g-force on the driver of the Cavalier was probably less than 1.7 g's (= 0.9 g/MPH x 1.9 MPH). This is about the same g-force that a person will experience by standing upright suddenly from a kitchen chair. Therefore, the driver of the Cavalier probably received a maximum g-force of less than 1.7 g's and the probability of physical injury was nil.

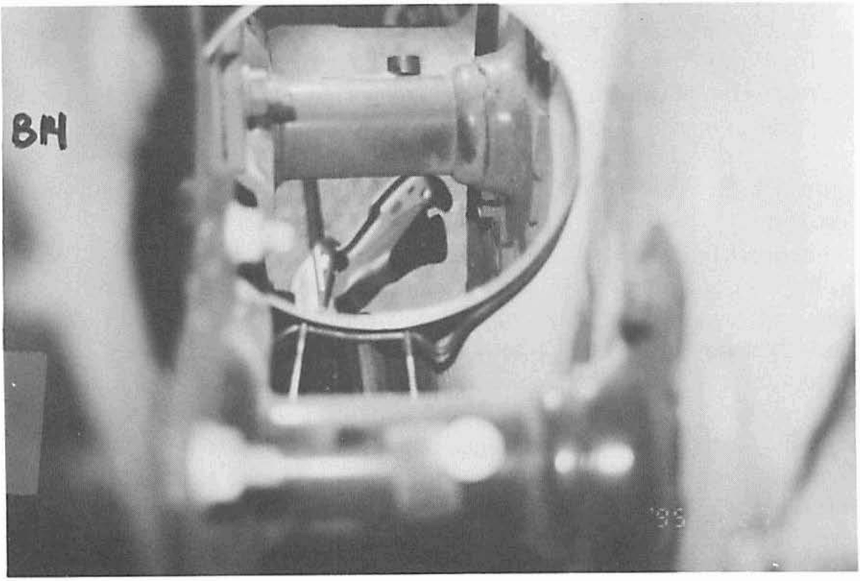
Approach II: Analytical Comparison of Damage With Published Crash Test Data

Federal Delta-V Requirements For A Bumper: Effective for 1973 models, the US Department of Transportation imposed the first federal bumper standards and required the protection of safety-related equipment like lights and hood latches during 5 MPH front-into-flat-barrier and 2.5 MPH rear-into-flat-barrier impacts.[9] These requirements went through several more-restrictive iterations until, effective for the 1980 models, federal standards required that all front and rear car bumpers withstand a 5 MPH flat-barrier impact with only minor cosmetic damage to the bumper and no damage to the body. Vehicles were successfully manufactured to meet that requirement and, in fact, the 1981 Ford Escort had the best bumpers the Insurance Institute for Highway Safety (IIHS) has ever tested.[10] However, effective for 1983 models, the National Highway Traffic Safety Administration (NHTSA) relaxed the federal requirements so that bumpers needed only to withstand a 2.5 MPH impact. Further, the damage criteria was also relaxed to allow unlimited bumper damage with no



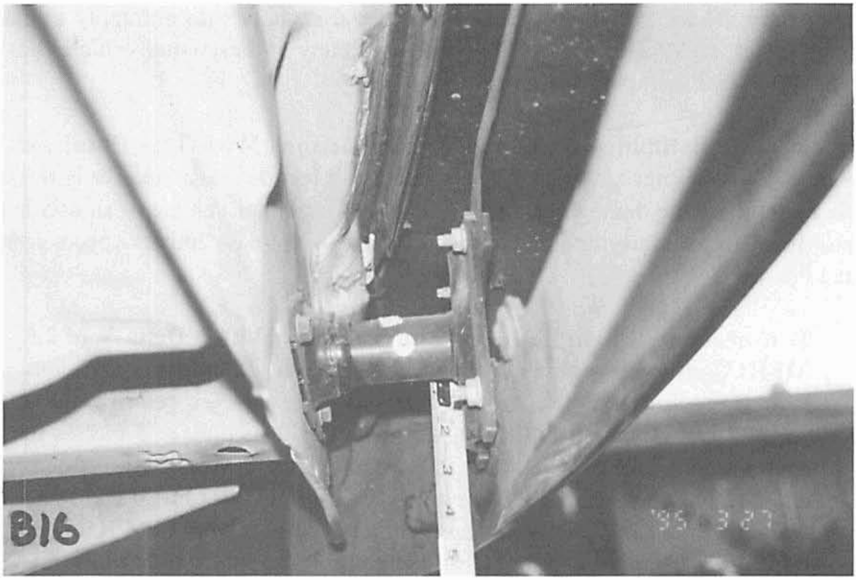
Photograph 1

B12: The EAD on the left side of the bumper shows no evidence of any stroke. The cylindrical magnet is 0.5" in diameter and the tape measure is 18" high.



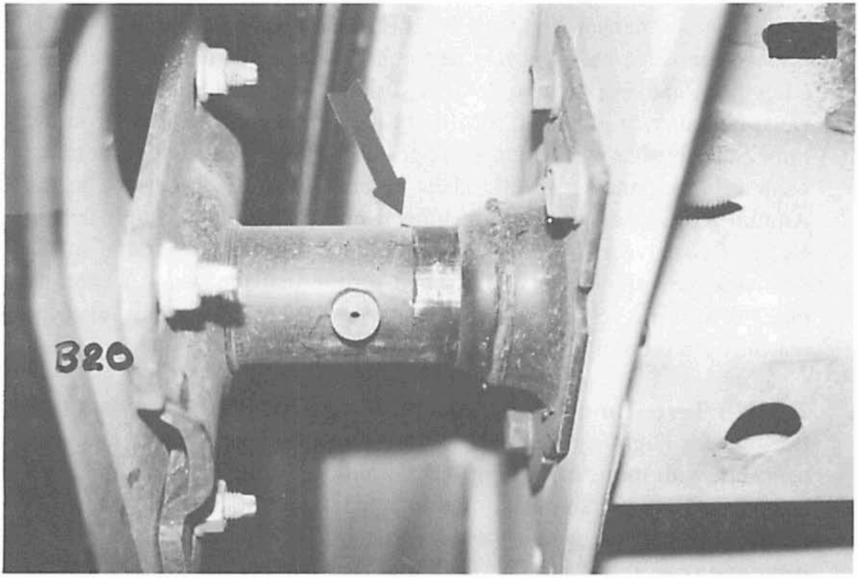
Photograph 2.

B14: The top surface of the left EAD is reflected in the mirror and, also, shows no evidence of any stroke. The EAD piston is 1.53" in outside diameter and 2.7" long.



Photograph 3

B16: The EAD on the right side of the bumper shows evidence of a stroke from the 11 to 5 o'clock positions, as viewed from the rear of the vehicle. The tape is 18" high.



Photograph 4

B20: The right EAD (viewed from 4 o'clock position) received a 0.5" long stroke at some time in the past. This EAD piston is 1.53" in outside diameter and 2.5" long.

damage to the car body. However, these federal standards do not apply to passenger vans, pickups or trucks. Consequently, many of these latter vehicles have relatively weak bumper assemblies.

Delta-V Estimates Based Upon Comparisons With Test Data: Since there is a significant amount of low-speed crash test data available, it is possible to compare the damage (or lack of damage) for either vehicle in an accident and, then, to estimate the impact speed or, the worst-case impact speed, from that comparison.

1. Consumers Union Performs Bumper Tests With a Delta-V of 2.5 MPH: Consumers Union (CU) tests the strength of bumpers on various vehicles each year and reports on the damage it finds. CU refers to these tests as their "bumper basher" tests.[11] The CU tests deliver 3 bashes to each bumper (one 5 MPH bash in the center, one 5 MPH bash off-center, and one 3 MPH bash at a corner) with a moving ram of the same weight as the car. The bashes are administered at a height of 20 inches on the front bumper and, to reflect the possible height of a braking vehicle, 16" on the rear bumper.

In terms of energy dissipation (see equations {13} and {18}), CU's 5 MPH bash with the weighted ram is equivalent to a 3.5 MPH impact against a solid barrier, and their 3 MPH bash is equivalent to a 2.1 MPH impact against a solid barrier. By conservation of linear momentum, CU's front and rear bash at 5 MPH with the weighted ram will cause a Delta-Velocity of about +2.5 MPH to the vehicle and -2.5 MPH to the ram. Such bashes will often damage the bumper and, sometimes, the vehicle body panels as well. Their test results are summarized in the Annual Auto Issue of the Consumer Reports magazine, published in April of each year. More detailed test data can often be found in the comparative vehicle tests performed throughout the year. A running index of specific vehicle tests during the prior 12 months are listed at the rear of each monthly "Consumers Reports" magazine.

2. IIHS Tests Bumpers at a Delta-V of 5.0 MPH: The Insurance Institute for Highway Safety (IIHS) performs bumper impact tests each year, but with more severe impacts than those of the Consumers Union tests. The IIHS tests consist of four impacts, all at 5 MPH against an immovable barrier, which yields a true Delta-Velocity of 5 MPH for front and rear impacts.[10] The front and rear bumpers are first impacted against a flat barrier (this test provides a relatively distributed load), then the front bumper is impacted at an angle into a flat barrier, and then the center of the rear bumper is impacted against a vertical pole (these latter two tests provide a relatively concentrated load). The impact between two actual vehicles will, generally, have a configuration somewhere between the concentrated load and distributed load, depending on the curvature of the two bumpers. The IIHS test results are published in their Status Report magazine, available free from IIHS at 1005 North Glebe Road, Arlington, VA 22201, telephone 703/247-1500 and Fax 703/247-1678.

3. Mitchell Guide Provides Parts Data For Different Years And For Sister Vehicles: Since test data cannot always be obtained for a specific vehicle, it may become necessary to rely on test data for a similar vehicle, like the same model but for a different year, or a sister vehicle from the same manufacturer. One of the series of Mitchell Collision Estimating Guides can be a useful resource to identify which vehicles have identical components and bumper assemblies. It is generally only necessary to verify that both vehicles have the same part numbers for each component of the bumper assembly.

4. Case in Point: Worst-Case g-Force from Comparisons With Published Test Data:

GMC Impacts Hyundai: A 2600#, 1986 Hyundai Excel was stopped behind traffic. Meanwhile, a 3500#, 1986 GMC Safari minivan, skidded on the pavement (covered with dirt and gravel from nearby construction activities) and struck the rear of the Hyundai. Sixteen months later an attorney requested an investigation to estimate the g-force on the occupants of the Hyundai.

The Evidence: Neither vehicle was available for personal inspection but both had been inspected and photographed by a professional appraiser. The appraiser reported that the GMC bumper had no damage but that the Hyundai bumper cover had two puncture marks on the right side that he attributed to the license plate mounting bolts on the GMC. The Hyundai brakes were probably not applied during or after the impact and the vehicles ended up 1.5' to 4.0' apart, depending upon which deposition you read.

The Analysis: The bumper heights on the GMC and Hyundai were determined from the AutoStats computer data base to be 18" and 22" high, respectively.[12] Using the standard 6" x 12" license on each vehicle as a guide, the puncture marks were scaled to be 8" apart and 16" high whereas the GMC license studs were only 7" apart and 10" high. Therefore, the marks were probably not caused by this accident.

Consumers Union had tested the bumpers on a 1986 Chevrolet Astro and gave the bumpers a very poor rating.[13] Their "bumper basher" tests at a Delta-V of 2.5 MPH mangled the bumpers, dented the fender and the body, and knocked off the lower splash shield. Their estimated cost was $497 to repair the front bumper. The Mitchell Collision Estimating Guide Domestic, General Motors, revealed that the GMC did not have an energy absorbing bumper system, and revealed that the bumper systems on all 1985 to 1991 Chevrolet Astros have the same part numbers as those for the GMC Safari. Therefore, the worst-case Delta-V that the GMC Safari could receive without damage was probably less than 2.5 MPH.

Consumer's Union had tested the bumpers on a 1986 Hyundai Excel and found that their "bumper basher" tests at a Delta-V of 2.5 MPH caused no damage to the front or rear bumpers.[14] However, IIHS tested the bumpers on a 1988 Hyundai Excel and found that their two impacts to the rear bumper at a Delta-V of 5 MPH caused $395 of damage.[15] The Mitchell Collision Estimating Guide Imported, Asian, revealed that all Hyundai Excels from 1986 to 1989 had identical parts. Considering the severity of the IIHS vertical pole impact test, a rear impact by another vehicle against the rear bumper of the Hyundai will probably not cause any damage up to a Delta-V of about 5 MPH.

The Key: Since the GMC did not receive any damage in this accident, but CU tests show it would probably be damaged by a 2.5 MPH impact, then the worst-case Delta-V that the GMC could receive without damage was probably less than 2.5 MPH. Then, by conservation of linear momentum, the worst-case Delta-V to the Hyundai was determined to be less than 3.4 MPH and the initial speed of the GMC would be less than 5.9 MPH. Based on these velocities, it was also found that the separation distance between the vehicles was reasonable. The rolling resistance of the Hyundai tires was about 0.1 and the coefficient of friction was about 0.4 to 0.7 on the gravel,[16] so the GMC would have skidded an additional 7" to 12" after impact and the vehicles would end up with a 2.8' to 3.3' separation, right in line with expected separation, as stated in the evidence.

The g-Force: The maximum g-force on the occupants in the Hyundai was probably less than 3.1 g's (= 0.9 g/MPH x 3.4 MPH). This is about the same g-force that a person will experience from a good uninhibited sneeze or a typical non-exaggerated cough. Therefore, the occupants of the Hyundai probably received a maximum g-force of less than 3.1 g's and the probability of physical injury was low.

Approach III: Analysis of Indirect Evidence

Engineering Analysis Based Upon Indirect Evidence from the Case: When neither vehicle in an accident has EADs or any physical evidence of an impact, and neither vehicle has exhibited any damage in CU's 2.5 MPH or IIHS's 5.0 MPH impact tests, then the forensic engineer will need to search for some other "indirect" evidence on which to base the analysis. That indirect evidence can usually be found in the file data (police data, depositions, witness statements, statements of the vehicle drivers or occupants, etc.) or observed at the accident site. The engineer can then base the analysis on that indirect evidence and develop a reasonable estimate of impact velocity. Often these simplified analyses will provide confirmation of results obtained by other analyses or from comparative crash test data.

1. Case in Point: g-Force Based on Idling Acceleration Capability of Vehicle:

Maxima Impacts Monte Carlo: A 4000#, 1985 Chevrolet Monte Carlo was stopped at a traffic light when it was impacted by a 3400#, 1988 Nissan Maxima. Nineteen months later a defense attorney requested an investigation to estimate the g-force on the occupants of the Monte Carlo.

The Key: In this case, the Monte Carlo had EADs so an analysis was performed to determine the Delta-V based on the stroke of the EADs. However, a secondary analysis was also performed to verify the EAD analysis. This case in point is related to the secondary analysis.

The Evidence: The Maxima had stopped about 3' to 5' behind the Monte Carlo. The Maxima driver then started looking at some papers on the passenger seat and did not realize the Maxima had started to idle forward. The Maxima impacted the rear of the Monte Carlo before the driver looked up.

The Analysis: An exemplary Maxima was used to perform a series of idling acceleration tests. Five timed tests were performed with idling accelerations over the 3' and 5' distances. The average acceleration times were 1.81 and 2.34 seconds so the average acceleration rate was 1.83 feet/second^2 over both distances. Therefore, the velocity of the Maxima at the end of the 3' and 5' acceleration distances would be 2.3 and 2.9 MPH, respectively. These values exactly bracketed the most likely 2.6 MPH impact speed that was determined by the EAD analysis. Therefore, by conservation of linear momentum, the Delta-V to the Monte Carlo was about 1.2 MPH while that to the Maxima was about -1.4 MPH.

The g-Force: The maximum g-force on the occupants in the Monte Carlo was most likely about 1.1 g's (= 0.9 g/MPH x 1.2 MPH). This is about the same g-force that a person will experience during a quick look over their shoulder as for a traffic check. Therefore, the occupants of the Monte Carlo probably received a maximum g-force of less than 1.1 g's and the probability of physical injury was nil.

2. Case in Point: g-Force Based on Maximum Acceleration Capability of Vehicle:

Buick Impacts Jeep: A 3100#, 1981 Jeep CJ-7 was stopped on the concrete pad at a gas station when it was impacted by a 2700#, 1991 Buick Skylark. Three months later an insurance claims representative requested an investigation to estimate the g-force on the driver of the Jeep.

The Evidence: The Buick had stopped about 4' to 6' behind the Jeep while the Jeep was filled with gas. As the Jeep driver prepared to depart, the Buick driver's foot slipped off the brake and onto the accelerator. The tires began to spin as the Buick accelerated forward and the driver was not able to withdraw his foot from the accelerator prior to impact. Inspection of the Jeep revealed some minor damage, including deformation of the steel structure that supported the spare tire. Inspection of the Buick was uncertain because the vehicle had been damaged extensively in a subsequent roll-over accident that, also, damaged the front grillwork and bumper of the Buick. Inspection of the accident site revealed that the concrete pad was washed with water frequently but was covered with oil spots.

The Key: An analysis, based upon the deformation of the Jeep's tire support structure was considered, but it was not certain that those deformations were created during this accident. Instead, the analysis was based upon the maximum acceleration capability of the Buick on the oil spotted concrete.

The Analysis: The maximum acceleration capability of the Buick, 11.6 FPS^2, was obtained from road test data in Consumer Reports magazine (6/92) for a Chevrolet Cavalier, a "sister" car to the Buick, with the same engine and transmission. The maximum acceleration capability was modified to account for the different weights of the two cars. Curb weights of the vehicles were obtained from Consumers Reports and the AutoStats computer data base. The coefficient of friction for the Buick's tires on the concrete pad was estimated to be 0.35 to 0.60, based on data from the Northwestern University Traffic Institute's Traffic Accident

Reconstruction Manual, and modified to account for the Buick's spinning tires on the oil-spotted concrete. The worst-case impact speed of the Buick was estimated to be 8.0 MPH with an acceleration time of 1.0 second. Then, by conservation of momentum, the worse-case Delta-V to the Jeep and Buick were determined to be 3.8 MPH and - 4.2 MPH, respectively.

The g-Force: The maximum g-force on the driver of the Jeep was probably less than 3.4 g's (= 0.9 g/MPH x 3.8 MPH). This is about the same g-force that a person will experience during a typical non-exaggerated cough. Therefore, the driver of the Jeep probably received a maximum g-force of less than 3.4 g's and the probability of physical injury was low.

3. Case in Point: g-Force Based on Typical Driver's Reaction Time: Previa Van Impacts Camry: A 4280# Toyota Previa Van was waiting behind a 3000#, 1986 Toyota Camry at a stop light. Both were preparing to make a right hand turn. The Camry pulled forward and stopped. The Previa then accelerated forward and impacted the Camry. One month later an insurance claims representative requested an investigation to estimate the g-force on the driver of the Camry.

The Evidence: Inspection of the Camry and Previa revealed that neither vehicle had any visible damage that could be attributed to this accident. Inspection of the accident site revealed that the right turn lane had a 40' radius with a 1.5% uphill grade. The Camry driver had started to accelerate around the curve but, then, stopped. The Previa driver had moved forward, waited for a traffic opening, and started accelerating around the curve – directly into the rear of the Camry. The Previa driver did not see the Camry before the impact.

The Key: Drivers preparing to make a right turn at this corner must look over their left shoulder to watch for an opening in the oncoming traffic flow. During inspection of this intersection, nearly all drivers were observed to start accelerating prior to their looking forward again. Therefore, drivers at that corner were timed with a stop watch and, on the average, accelerated for 1.8 seconds before looking ahead. Therefore, the average time that the Previa would have accelerated before the driver looked forward was also estimated to be about 1.8 seconds.

The Analysis: The maximum acceleration capability of the Previa, 10.1 FPS^2, was obtained by plotting the low speed road test data from Consumer Reports magazine (10/92) and, then, modifying that data to account for the different weights of the vehicles. However, the Previa held a family on a leisurely outing so a "normal" acceleration rate of 4.9 FPS^2 was also considered as a more likely acceleration rate. With the acceleration time of 1.8 seconds, the nominal and worst-case impact speed of the Previa was estimated to be 6.0 and 12.4 MPH, respectively. Then, by conservation of momentum, the nominal Delta-V to the Camry and Previa would be 3.5 MPH and -2.5 MPH, respectively, and the worse-case Delta-V to the Camry and Previa would be 7.3 MPH and -5.1 MPH, respectively. Since Consumers Union "bumper basher" tests revealed that the Camry's bumper sustained no damage while the Previa's front bumper sustained $537 of damage, and because the normal acceleration rate would be more logical for the family activity, and for other reasons (too long to discuss for this case in point), the nominal acceleration rate was more likely than the maximum acceleration rate.

The g-Force: The worst-case g-force on the driver of the Camry was estimated to be 6.6 g's (= 0.9 g/MPH x 7.3 MPH) for which the probability of physical injury would be fair. However, based upon engineering judgment, it is more likely that the g-force was about 3.2 g's (= 0.9 g/MPH x 3.5 MPH). Therefore, the driver of the Camry, most likely, received a maximum g-force about 3.2 g's and the probability of physical injury was low.

Conclusion

Low-speed rear-end collisions can, generally, be analyzed to estimate the Delta-V of a vehicle, and the g-force on the occupants can then be estimated, even when there is only minor or, possibly, no visible damage to the vehicles. The Delta-V to a vehicle can generally be estimated by comparing the impact damage, or lack of impact damage, with published low-speed crash test data or, in many cases, by direct engineering analysis of the damaged mechanical components. In particular, this paper presents a method to estimate the worst-case Delta-V for a vehicle equipped with viscous-damper type of Energy Absorbing Devices (EADs) wherein the compression force is related to the rate of compression rather than to some displacement during compression. This type of EAD is common on General Motors vehicles. Once the Delta-V of a vehicle has been estimated, the g-force and probability of injury to the occupants can be estimated, based on published test data for instrumented human test subjects during such collisions.

Appendix Notation

Symbols:

Δ	Delta symbol, designates the "change of" for any quantity that follows the symbol.
ε	coefficient of restitution [dimensionless]
∞	symbol that designates "infinite" or "infinity" [dimensionless]
#	Pound symbol [pounds]
F	Force [pounds]
g	Gravitational acceleration constant [32.2 feet/second^2]
KE	Kinetic Energy [foot-pounds, or inch-pounds]
l	script "l", used to designate length of a piston stroke [inches]
V	Velocity or speed of a vehicle [MPH (Miles Per Hour) or FPS (Feet Per Second)]
W	Weight of a vehicle [pounds]

Subscripts:

$_0$	Zero, used to designate an "original" or initial condition
$_B$	Bullet vehicle, used to designate the vehicle which impacts another "target" vehicle
$_D$	Design, used to designate the value of a design parameter
$_T$	Target vehicle, used to designate the vehicle which is impacted by the "bullet" vehicle

Superscript:

"	used to denote "inches" of length
	used to denote "feet" of length and, also, to designate the velocity or speed of a vehicle after an impact.

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